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THE IMPACT OF GROUP VELOCITY DISPERSION ON FEMTOSECOND LASER FILAMENTATION WITH HIGHER-ORDER KERR EFFECT AT DIFFERENT ATMOSPHERIC PRESSURES

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The impact of group velocity dispersion (GVD) on femtosecond laser filamentation with the higher-order Kerr effect is studied at different atmospheric pressures. The results show that GVD makes the collapse distance to not meet the semi-empirical formula proposed by Dawes and Marburger, suppresses multiple focusing and splitting in time during the propagation process, and reduces the length of filament. In addition, we also compared the results with those considering only the third-order Kerr effect. This provides some information for the study of whether the defocus effect in the femtosecond laser filamentation is the higher-order Kerr effect or the plasma effect.

Keywords: femtosecond, filamentation, GVD, higher-order Kerr effect, pressure

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1. Introduction

For a long time, it is believed that the intense femtosecond laser pulse is not suitable for long-distance transmission in gas due to the linear effects such as the beam diffraction effect and the group velocity dispersion (GVD) effect. However, Braun et al. got the opposite result when they used the intense infrared femtosecond pulse to do experiments [1]. They found that the pulse intensity did not decrease after propagating for a distance, but increased. At about 10 m from the laser output end, the air molecules were even ionized and formed a bright plasma channel nearly 20 m long, which shows that the femtosecond laser pulse does not have dispersion in this region and maintains high intensity transmission. This is the first time to observe the laser filamentation phenomenon in air. After that, Nibbering and Fontaine et al. even observed filaments with a length of more than 50 m in optical experiments [2, 3].

According to the results of the above experimental observations, it has been known that whether the intense femtosecond laser can be transmitted in the gas medium for a long distance depends on the formation of the filament, and the laser filament contains a variety of physical processes, such as the nonlinear Kerr effect, multi-photon ionization, plasma defocusing, etc. Now there are two different views on the internal physical mechanism of filament formation. One is the classical model, which believes that the generation of filaments is due to the existence of third-order nonlinear Kerr effect, which makes the laser pulse produce nonlinear self-focusing. At the same time, the intense laser pulse ionizes the air to produce plasma, and the plasma has the defocusing effect on the laser. When the third-order nonlinear self-focusing effect and the plasma defocusing effect interact and reach the dynamic equilibrium, the laser beam can form a long plasma channel (i.e. filament) in air, which makes the transmission of a strong laser in the atmosphere reach a stable state. Many scholars have always recognized this view and have done a lot of research work [4–8]. After Loriot et al. measured the high-order Kerr coefficients in N_2, O_2, argon and air [9, 10], people turned their attention to the high-order Kerr effect. Béjot et al.

considered the higher-order Kerr effect (HOK model) in the numerical simulation of the intense femtosecond laser propagation in the gas. They found that the higher-order terms of the nonlinear refractive index are in the dominant position (n_2 and n_6 are positive focusing, n_4 and n_8 are negative defocusing), that is different from the defocusing role of plasma in the classical model, which shows that plasma is not a necessary condition for the generation of femtosecond filaments [11]. This new theoretical model of filamentation quickly attracted the interest of many scholars [12–19]. Loriot et al. found that plasma defocusing mainly acts on short wavelength and long pulse width laser pulses, while the higher-order Kerr effect mainly acts on long wavelength and narrow pulse width laser pulses [13], and Béjot et al. verified this conclusion through experiments [15]. In addition, when using the HOK model to simulate the evolution of the internal peak intensity of the filament in femtosecond laser argon [16], it was found that the phenomena such as pulse self-compression [20] and pulse splitting [21] would not occur, which is inconsistent with the experimental results. However, using the classical model, a good fitting result can be obtained. It can be seen that the contribution of higher-order nonlinear Kerr effect to the refractive index in argon is overestimated, and plasma still plays a major role in the defocusing process. Therefore, there is still a certain degree of controversy about whether the plasma or the n_4 and n_8 terms in the higher-order Kerr effect play a defocusing role, and so far there is no final conclusion.

Whether in the classical model or the HOK model, laser filamentation is caused by the interaction of nonlinear effects. Researchers rarely consider linear effects, such as GVD. This is because GVD is too small in gas, it cannot play an important role in preventing laser collapse [22]. But when investigators began to pay attention to the influence of atmospheric pressure on laser filamentation [23–27], some studies found that the higher the atmospheric pressure, the greater the influence of GVD on laser filamentation [28]. However, up to now, there are some disputes about whether to use the classical model or the HOK model, so it is appropriate to use the HOK model to study the influence of GVD on the femtosecond laser filamentation at different atmospheric pressures.

In this paper, we will use the HOK model to simulate the propagation process of femtosecond laser in the atmosphere, study the influence of GVD on laser filamentation under different atmospheric pressure and compare the results with the classical model. The subsequent structure of this paper is as follows: the second section introduces the nonlinear Schrödinger equation (NLSE) of intense femtosecond laser propagation in the atmosphere, the third section gives the results of numerical simulation, and the fourth section gives conclusions.

2. Propagation equation

The propagation equation of the intense femtosecond laser in air can be described by the nonlinear Schrödinger equation (NLSE) including linear and nonlinear effects. With the z axis as the propagation direction, the 2D+1 propagation equation can be written as follows [18, 29, 30]:

$$\frac{\partial E}{\partial z} = -\frac{\mathrm{i}k''}{2}\frac{\partial^2 E}{\partial t^2} + \frac{\mathrm{i}}{2k_0}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)E + \frac{\mathrm{i}k_0}{n_0}\Delta N_{\mathrm{Kerr}} E - \frac{\mathrm{i}k_0}{2}\frac{\omega^2}{\omega_0^2}E - \frac{\beta^{(K)}}{2}|E|^{2K-2}E, \quad (1)$$

$$\frac{\partial \rho}{\partial t} = \frac{\beta^{(K)}}{K\hbar\omega_0}|E|^{2K}\left(1 - \frac{\rho}{\rho_{at}}\right). \quad (2)$$

Here E is the electric field, the laser intensity $I = |E|^2$, $k_0 = 2\pi n_0/\lambda_0$, $\omega_0 = 2\pi c/\lambda_0$, λ_0 is the wavelength, k'' is the second-order dispersion coefficient, $\omega^2 = \rho e^2/m_e\varepsilon_0$ (ρ, m_e, e are the electron density, electron mass and electron charge, respectively), ρ_{at} is the neutral atoms density, $\beta^{(K)}$ is the nonlinear coefficient for K-photon absorption [31], and ΔN_{Kerr} is the nonlinear Kerr effect term. When the delayed Kerr effect is considered, ΔN_{Kerr} can be expressed as [11, 32, 33]

- classical model: $\Delta N_{\mathrm{Kerr}} = (1 - f)n_2|E|^2 + fn_2 \int_{-\infty}^{t} R(t - t')|E(t')|^2 \mathrm{d}t'$,
- HOK model: $\Delta N_{\mathrm{Kerr}} = (1 - f)n_2|E|^2 + \sum_{j=2}^{4} n_{2*j}|E|^{2*j} + fn_2 \int_{-\infty}^{t} R(t - t')|E(t')|^2 \mathrm{d}t'$,

where $R(t)$ $\frac{1}{\tau_K}\mathrm{e}^{-t/\tau_K}$ is the time-retarded response of medium. We input the initial electric field $E(z = 0, r, t) = E_0\exp(-2r^2/r_0^2 - 2t^2/\tau^2)$, $E_0 = \sqrt{\frac{2P_{\mathrm{in}}}{\pi r_0^2}}$, the pulse duration $\tau = 100$ fs, and

r = 1 mm is the beam waist. The critical power $P_{cr} = 3.77\lambda^2_0/8\pi n_0 n_2$, and $P_{in} = 4P_{cr}$. The parameter value used in this paper is shown in Table 1. P is the ratio of pressure to standard atmospheric pressure [34].

In the numerical simulation of this paper, we mainly consider three atmospheric pressures: 1, 2 and 5 atm, namely P = 1, 2 and 5.

Table 1. The parameter values.

Parameter	Value
λ_0 (nm)	800
k'' (fs^{2}cm^{-1})	0.2P
K	10
$\beta^{(K)}$ (cm^{-17}W^{-9})	1.27$P \times 10^{-126}$
ρ_{at}	2.7$P \times 10^{25}$
n_2 (cm^{2}W^{-1}) [9]	1.2$P \times 10^{-19}$
n_4 (cm^{4}W^{-2})	–1.5$P \times 10^{-33}$
n_6 (cm^{6}W^{-3})	2.1$P \times 10^{-46}$
n_8 (cm^{8}W^{-4})	–0.8$P \times 10^{-59}$
τ_K (fs) [33]	70
f [35]	0.5

3. Results and discussion

Figure 1 shows the evolution of the on-axis intensity, plasma density and the beam radius of laser pulse with the propagation distance z at 1, 2 and 5 atm in two cases, with and without GVD, for the classical model (three rows above) and the HOK model (three rows below). It can be seen from Fig. 1 that in the classical model the clamping intensity basically does not change with the increase of atmospheric pressure, the beam radius is inversely proportional to atmospheric pressure, and the plasma density is proportional to atmospheric pressure. The above results have no relationship with whether to consider GVD. However, GVD has a great impact on L_c (L_c is the collapse distance). GVD makes L_c to meet the semi-empirical formula [31] no longer, but increases with the increase of atmospheric pressure. These results are consistent with the previous results [28]. In the HOK model, we observed the same results as in the classical model. Hence, GVD has the same effect on the on-axis intensity, plasma density, beam radius and L_c in both the HOK model and the classical model.

Figure 2 shows the evolution of the time profiles of the laser pulse with the propagation distance z at different atmospheric pressures in two cases, with and without GVD, for the classical model (two rows above) and the HOK model (two rows below). It can be seen from Fig. 2 that in the classical model the same results of the cases, with GVD and without GVD, are the following: the laser pulse splits in time, a 'dip' structure appears [36], and the 'dip' width decreases with the increase of atmospheric pressure. The difference is that in the case without GVD the laser pulse will split again in time at 'dip', while in the case with GVD it will not. Therefore, the GVD will inhibit the time splitting of laser pulse at 'dip'. In the HOK model, the laser pulse will split in time whether GVD exists or not, and in the case without GVD, the laser pulse will undergo multiple focusing and splitting with time in the propagation direction. Therefore, the impact of GVD on the laser filamentation in the HOK model is to suppress multiple focusing and splitting in time during the propagation process, which is different from the impact of GVD on the laser filamentation in the classicl model.

Figure 3 shows the evolution of the radial profiles of the laser pulse with the propagation distance z at different atmospheric pressures in two cases, with and without GVD, for the classical model (two rows above) and the HOK model (two rows below). As can be seen from Fig. 3, whether in the classical model or the HOK model, when considering GVD, the laser pulse can only form a short filament in the atmosphere, and this phenomenon becomes more obvious with the increase of atmospheric pressure. When GVD is not considered, a very long filament can be observed in both the classical model and the HOK model. The only difference is that continuous and stable filaments are formed in the HOK model, while discontinuous filaments are formed in the classical model. This is because the laser filamentation in the HOK model mainly depends on the higher-order Kerr effect, which does not change with atmospheric pressure and propagation distance, and the light intensity is relatively stable. For the classical model, the defocusing effect mainly depends on the plasma, and its density depends on the propagation distance and atmospheric pressure, which leads to the instability of light intensity, so there is a discontinuous filament.

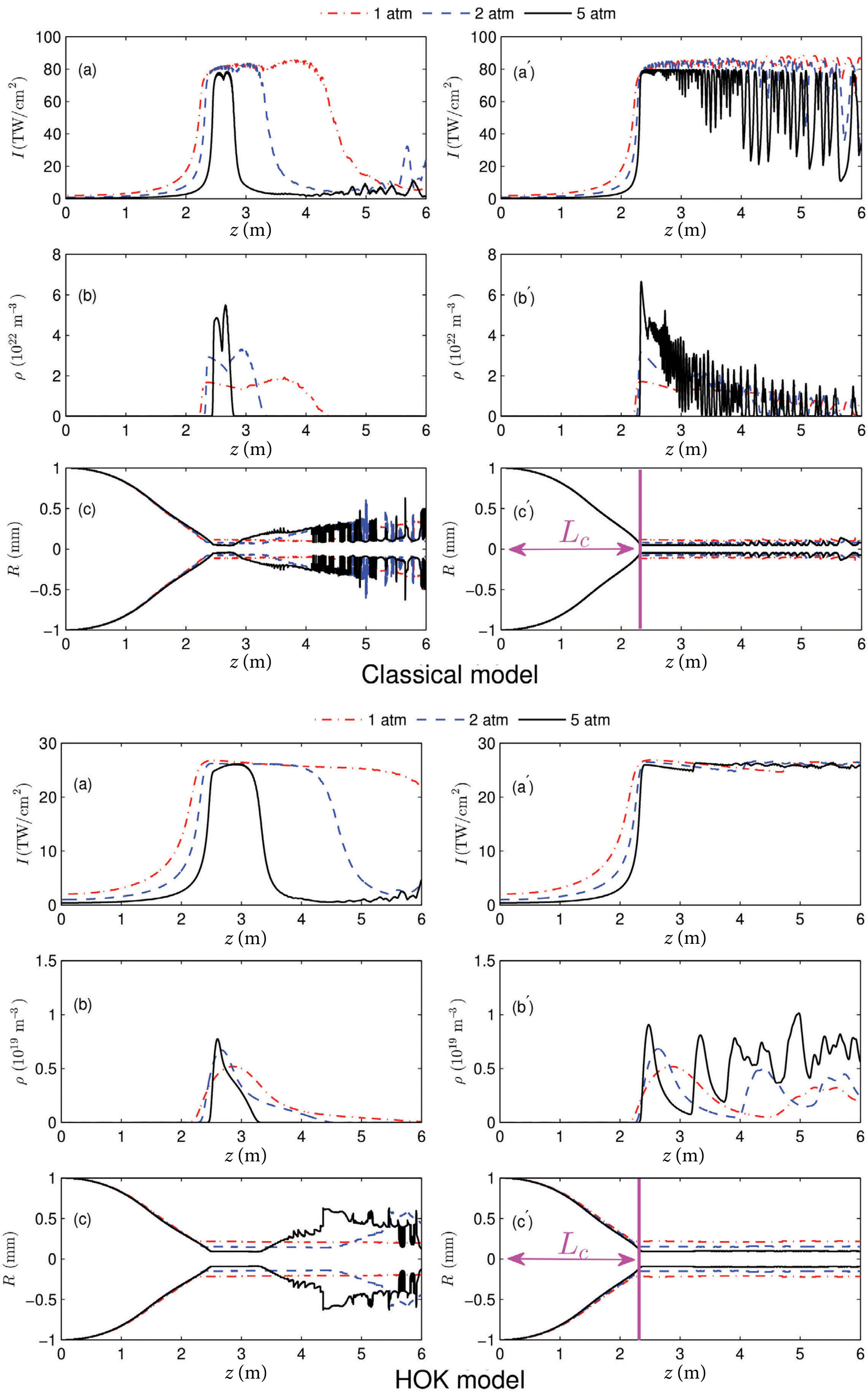

Fig. 1. Evolution of the on-axis intensity (a, a′), plasma density (b, b′) and the beam radius (c, c′) of the laser pulse with the propagation distance z at different atmospheric pressures in two cases, with (a–c) and without (a′–c′) GVD, for the classical model (three rows above) and the HOK model (three rows below).

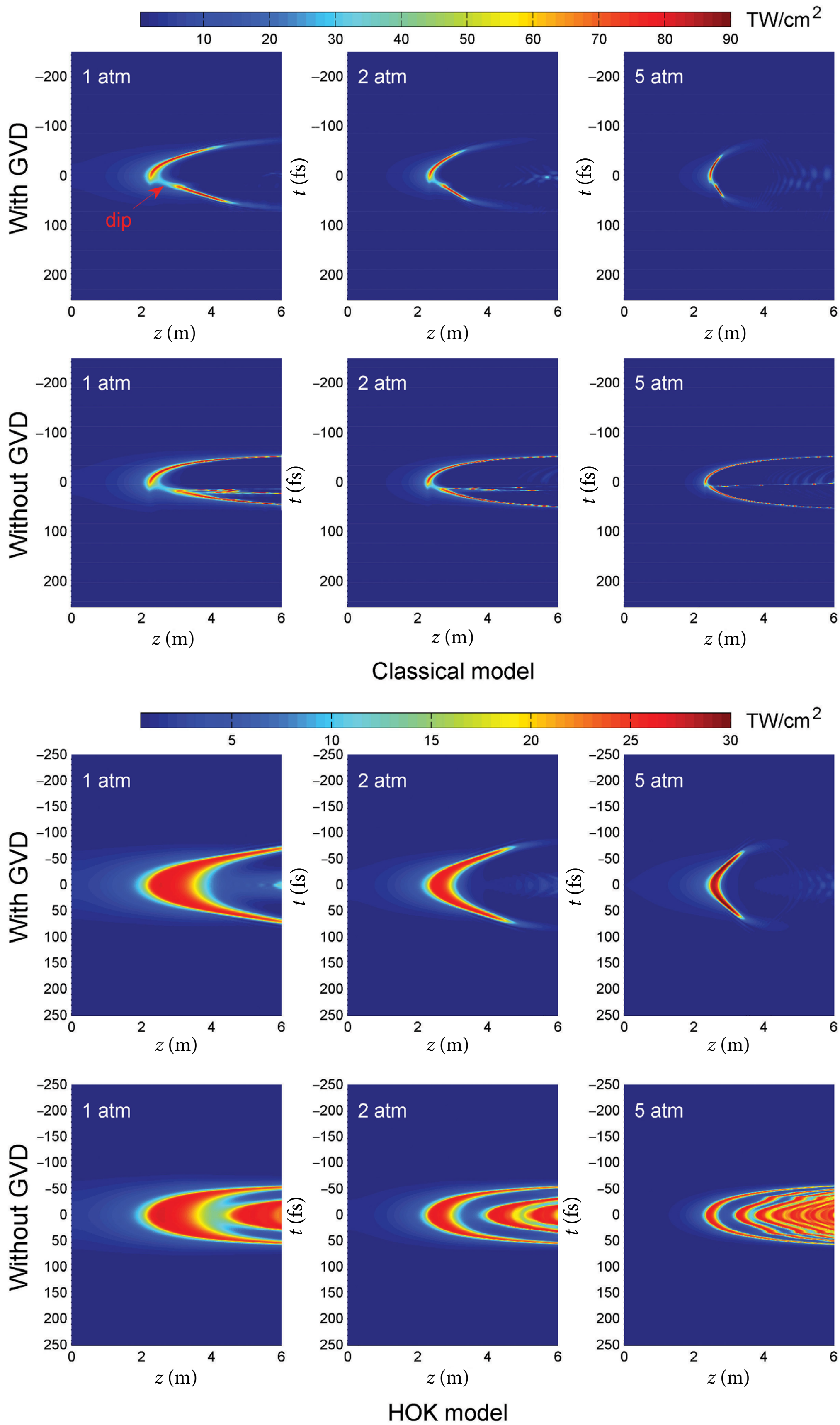

Fig. 2. Evolution of the time profiles of the laser pulse with the propagation distance z at different atmospheric pressures in two cases, with and without GVD, for the classical model (two rows above) and the HOK model (two rows below).

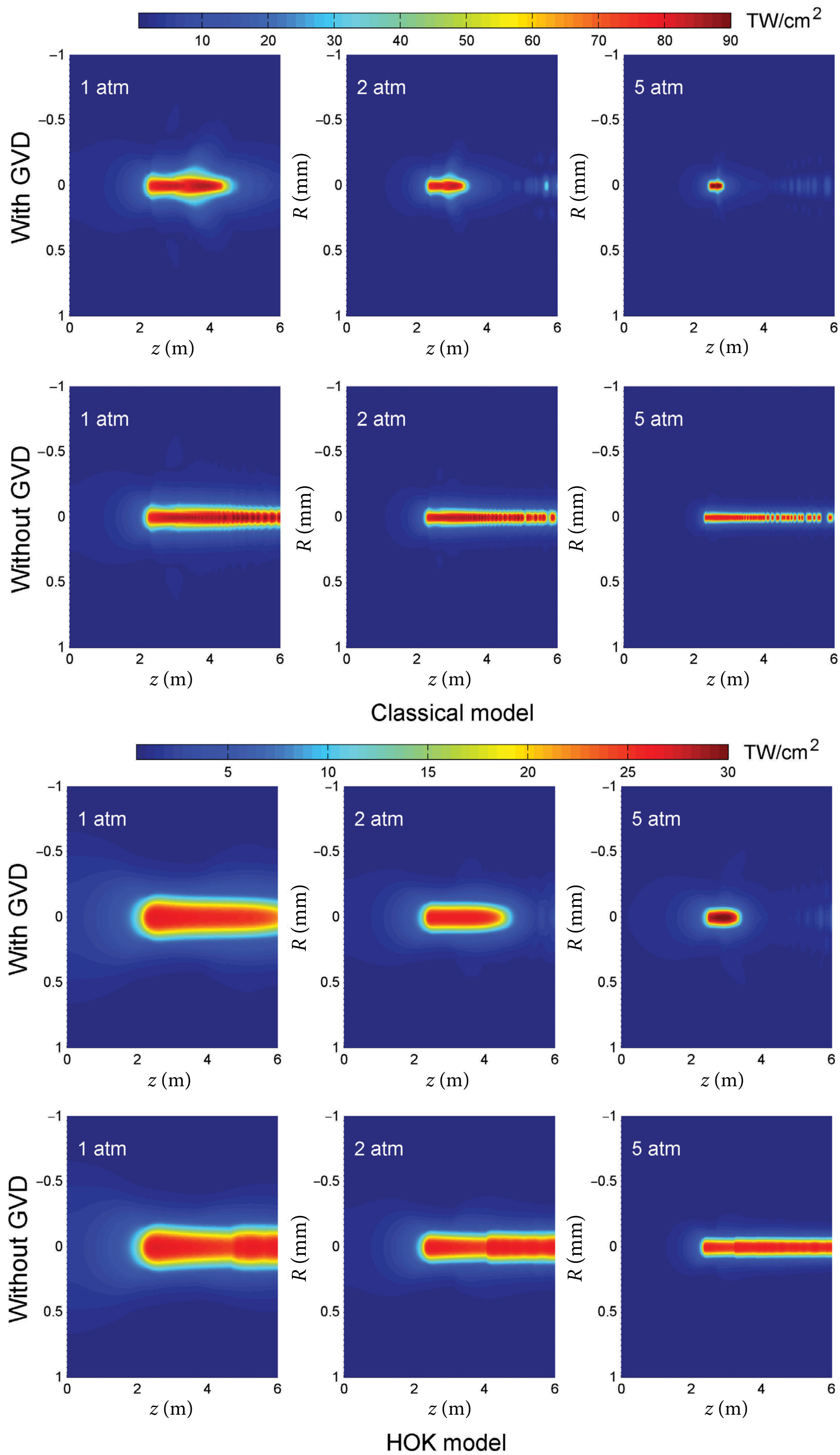

Fig. 3. Evolution of the radial profiles of the laser pulse with the propagation distance z at different atmospheric pressures in two cases, with and without GVD, for the classical model (two rows above) and the HOK model (two rows below).

4. Conclusions

In this paper, the classical model and the HOK model are used to numerically simulate the influence of GVD on femtosecond laser filamentation. The on-axis intensity, plasma density, beam radius, collapse distance L_c, the time profile and radial profile of the laser pulse are studied, respectively, and compared with the results of numerical simulation using the classical model. The following conclusions are obtained:

(1) In the HOK model, GVD has no effect on the relationship between three aspects (clamping intensity, plasma density and beam radius) and atmospheric pressure, but only on the relationship between L_c and atmospheric pressure. When GVD exists, L_c increases with the increase of atmospheric pressure, which does not conform to the semi-empirical formula. These results are basically the same as those of the classical model.

(2) In the HOK model, GVD suppresses multiple focusing and splitting in time during the propagation process, while in the classical model, GVD suppresses the time splitting of laser pulse at 'dip'.

(3) In the classical model and the HOK model, GVD reduces the length of filament. However, in the HOK model, GVD reduces continuous and stable filaments, while in the classical model, GVD reduces discontinuous and unstable filaments. Through this study, we give the difference and connection between the classical model and the HOK model to study the influence of GVD on laser filamentation, which will provide a theoretical basis for the correct model to study laser filamentation in the future.

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GRUPINIO GREIČIO DISPERSIJOS ĮTAKA FEMTOSEKUNDINIO LAZERIO SU AUKŠTESNIOSIOS EILĖS KERO EFEKTU FILAMENTACIJAI ĮVAIRAUS SLĖGIO ATMOSFEROJE

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