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CANONICALLY QUANTIZED SOLITON IN THE BOUND STATE APPROACH TO HEAVY BARYONS IN THE SKYRME MODEL

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The bound state extension of Skyrme's topological soliton model for the heavy baryons is quantized canonically in arbitrary irreducible representations of the SU(3) flavour group. The canonical quantization leads to an additional negative mass term, which stabilizes the quantized soliton solution. The heavy flavour meson in the field of the soliton is treated with semiclassical quantization. The representation dependence of the calculated mass spectra for the strange, charm, and bottom baryons is explored and compared to the existent empirical spectra.

Keywords: Skyrme model, baryon mass spectra, heavy baryons

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1. Introduction

Skyrme's topological soliton model for the baryons [1] is a chiral symmetric mesonic representation of QCD in the limit of large colour number N_c. In this limit baryons are constructed as topological soliton solutions to the effective chiral meson Lagrangian. The baryon number is the second Chern number of the meson field [2]. In the semiclassical version of the model the mass of the pion, which is the Goldstone boson of the spontaneously broken chiral symmetry, is introduced through adding a complementary symmetry breaking term to the Skyrme Lagrangian.

The model yields a qualitative description of the lower energy part of the spectra of the nucleons and delta resonances, but its direct extension to describe strange, or heavier, baryons fails phenomenologically because of the badly broken SU(3) symmetry [3]. In the chiral limit all quarks are massless, which implies a model with unbroken SU(3) and higher flavour symmetry groups. The reason is that the flavour quantum numbers are associated with rotations of collective coordinates, although the collective coordinates approach is associated with unbroken flavour symmetry. When the symmetry is badly broken, the rigid rotator treatment fails, with poor phenomenology as a consequence [4]. A phenomenologically more successful alternative approach of treating hyperons as bound states of Skyrme solitons and non-topological fields of K mesons was introduced by Callan and Klebanov [5, 6]. In this approach the hyperons with strange and heavier flavour number appear as bound states of mesons with the appropriate quantum number and the soliton. In the harmonic approximation the meson–soliton system is described by a linear wave equation.

When the meson field is expanded perturbatively the model Lagrangian splits into two parts corresponding to soliton and meson field, which interacts with the soliton field. We show that canonical quantization of the soliton, which respects the non-commutativity of quantum variables, leads to quantum stabilizing term that lowers the soliton mass. The dependence of the meson–soliton interaction on the representation of the heavy flavour SU(3) is analysed here in detail, with emphasis on the Wess–Zumino and the symmetry breaking terms. It is found that new bound states with higher excitation number can appear in higher representations of SU(3) group, which are absent in the fundamental representation. In general the ordering of the calculated spectra of the heavy flavour baryons agree with the existent empirical values.

The model Lagrangian and the traditional treatment of the kaon fields that represent heavier meson fields in bound state approach is reviewed in Sec. 2. The generalization of the bound state Skyrme model to representations of arbitrary dimension is also presented in this section. In Sec. 3 the soliton is quantized canonically *ab initio* in the collective coordinates framework and

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embedded into the bound-state model Lagrangian. Section 4 contains the wave equation for the meson field in arbitrary representations and the diagonalization of the Hamiltonian density in terms of creation-annihilation operators to derive the final model Hamiltonian. Section 5 contains physical interpretation of the hyperon states that arise in the model, along with comparison of calculated spectra to the experimental data. Section 6 contains a summarizing discussion. The notation employed for the SU(3) group algebra is given in Appendix.

2. Model Lagrangian

2.1. The bound state model

The action of Skyrme model extension to the hyperons is

$$S = \int \mathrm{d}^4x \left(\mathcal{L}_{\mathrm{Sk}} + \mathcal{L}_{\mathrm{SB}}\right) + S_{\mathrm{WZ}} . \quad (1)$$

Here $\mathcal{L}_{\mathrm{Sk}}$ is Lagrangian density of the original Skyrme model [1],

$$\mathcal{L}_{\mathrm{Sk}} = -\frac{f_\pi^2}{4}\mathrm{Tr}\left\{(\partial_\mu \mathbf{U})\mathbf{U}^\dagger(\partial^\mu \mathbf{U})\mathbf{U}^\dagger\right\} + \frac{1}{32e^2}\mathrm{Tr}\left\{\left[(\partial_\mu \mathbf{U})\mathbf{U}^\dagger, (\partial_\nu \mathbf{U})\mathbf{U}^\dagger\right]^2\right\}, \quad (2)$$

where f_π is pion decay constant, e is model parameter, and $\mathcal{L}_{\mathrm{SB}}$ and S_{WZ} are the actions for the chiral symmetry breaking and the Wess–Zumino terms respectively.

In the limit of unbroken SU(3) symmetry the meson field $\mathbf{U}$ takes values in the SU(3) group algebra. When SU(3) symmetry is badly broken due to a large meson mass term, the physically relevant field configurations will be small fluctuations into heavy flavour directions around the pionic soliton.

Here we employ the chiral symmetric field ansatz [5]

$$\mathbf{U} = \sqrt{\mathbf{U}_\pi}\mathbf{U}_K\sqrt{\mathbf{U}_\pi} \quad (3)$$

to construct bound states, where

$$\sqrt{\mathbf{U}_\pi} = \exp\left(\mathrm{i}\frac{1}{f_\pi}\left(\hat{J}^{(1,1)}_{(0,1,m)}\hat{x}^{(m)}\right)F(r)\right), \quad (4)$$

$$\mathbf{U}_K = \exp\left(\mathrm{i}\frac{1}{f_K}\hat{J}^{(1,1)}_{(z,1/2,m)}\hat{K}_{(z,1/2,m)}\right). \quad (5)$$

Here f_K is kaon decay constant and the parameters (z, j, m) are used for the basis state notation for the canonical chain SU(2) $\subset$ SU(3) [5]. To a first approximation it suffices to expand the fields only up to the second order. The contribution of higher order terms, which describe self-interactions in the meson field, has been found to be small [7].

The kaon (the generalization to the D and B mesons is given in Refs. [8, 9]) field has the conventional isodoublet structure:

$$K^\dagger = \left(K^- \; \tilde{K}^0\right), \qquad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}. \quad (6)$$

The meson state is a field that takes values in the adjoint representation of the SU(3) algebra:

$$\begin{aligned} K^- &= \hat{K}_{(\frac{1}{2},\frac{1}{2},-\frac{1}{2})}, & K^+ &= \hat{K}_{(-\frac{1}{2},\frac{1}{2},\frac{1}{2})}, \\ \tilde{K}^0 &= -\hat{K}_{(\frac{1}{2},\frac{1}{2},\frac{1}{2})}, & K^0 &= \hat{K}_{(-\frac{1}{2},\frac{1}{2},-\frac{1}{2})}. \end{aligned} \quad (7)$$

The invariance of the soliton under combined spatial and isospin rotations implies the following condition for the eigenmode expansion of the meson field:

$$K(\mathbf{r}, t) = k(r, t)\; Y_{u\,l\,u_0}(\vartheta, \varphi). \quad (8)$$

Here l and u represent orbital and total angular momentum of the meson field respectively.

The radial part $k(r, t)$ of the meson field can be expressed in terms of energy eigenvalues as follows:

$$k(r, t) = \sum_{n>0}\left[k_{-n}(r)\,\mathrm{e}^{\mathrm{i}\omega_{-n}t}b_n^\dagger + k_n(r)\,\mathrm{e}^{-\mathrm{i}\omega_n t}a_n\right]. \quad (9)$$

Here a_n is the annihilation operator for state with strangeness $S = -1$ and $b_n^\dagger$ is the creation operator for state with strangeness $S = 1$.

2.2. Generalization to arbitrary representations

The pionic field may be expressed in any irreducible SU(3) group representation as direct sum of irreducible SU(2) group representations [10]:

$$\sqrt{\mathbf{U}_\pi} = \sum_{z,j}^{(\lambda,\mu)} \oplus D^j(\alpha). \quad (10)$$

Using the Maurer–Cartan form notation the explicit form of right chiral current of pionic field can be expressed as

$$\left(\nabla_k\sqrt{\mathbf{U}_\pi}\right)\sqrt{\mathbf{U}_\pi}^\dagger = \left\{\mathrm{i}\left(F' - \frac{1}{r}\sin F\right)(-1)^m\hat{x}_{-m}\hat{x}_k + \mathrm{i}\frac{1}{r}\sin F\,\delta_{m,k} - \frac{2\sqrt{2}}{r}\sin^2\frac{F}{2}\begin{bmatrix} 1 & 1 & 1 \\ -m & k & n \end{bmatrix}(-1)^m\hat{x}_n\right\} \times \left\langle\left|\hat{J}^{(1,1)}_{(0,1,m)}\right|\right\rangle. \quad (11)$$

After substitution of the bound state ansatz (3) into (2) and using (11) for the pion field and (5) for the meson field the Skyrme Lagrangian for the meson field in a $u = \frac{1}{2}$ state takes the form below after division by the factor

$$N = \frac{1}{4}\dim(\lambda,\mu)\, C_2(\lambda,\mu)\ : \tag{12}$$

$$L_{\text{Sk}} = \frac{1}{4f_K^2}\frac{f_\pi}{e}\int \mathrm{d}\tilde r\, \tilde r^2 \left(f\,\dot{\tilde k}^2 - h\,\tilde k'^2 + V_{\text{eff}}\,\tilde k^2\right) - M_{\text{cl}}\,. \tag{13}$$

Here M_{cl} is the classical soliton mass:

$$M_{\text{cl}} = 4\pi\frac{f_\pi}{e}\int \tilde r^2 \mathrm{d}\tilde r\, \frac{1}{2}[d + 2s + s(2d+s)]\,. \tag{14}$$

The coefficients in this expression are:

$$h = 1 + \frac{1}{4}2s,$$

$$f = 1 + \frac{1}{4}\,(d+2s)\,,$$

$$V_{\text{eff}} = -\frac{1}{4}\,(d+2s) - \frac{1}{4}2s(2d+s) + \frac{1}{\tilde r^2}\left[1 + \frac{1}{4}(d+s)\right]\left[2c^2 + (1-4c)l(l+1)\right] + \frac{1}{4}\frac{6}{\tilde r^2}\left\{s\left[c^2 - (2c-1)l(l+1)\right] + \frac{\mathrm{d}}{\mathrm{d}\tilde r}\left[\big(c - l(l+1)\big)\tilde F'\sin\tilde F\right]\right\}. \tag{15}$$

Here the standard notations employed are: $s = (1/\tilde r^2)\sin^2\tilde F$, $d = \tilde F'^2$, $c = \sin^2(\tilde F/2)$. The dimensionless parameters are: $\tilde r = ef_\pi r$, $\tilde F \equiv F(\tilde r)$, and $\tilde k \equiv k(\tilde r, t)$.

2.3. The Wess–Zumino term

The Wess–Zumino term plays crucial role in bound-state model. It involves the meson field only with one time derivative. This term splits the energies of states with strangeness $S = -1$ and $S = 1$ one from another. The Wess–Zumino action is

$$S_{\text{WZ}} = -\frac{\mathrm{i}N_c}{240\pi^2}\int \mathrm{d}^5x\, \epsilon^{\mu\nu\alpha\beta\gamma}\,\mathrm{Tr}\,\{\mathbf{R}_\mu\mathbf{R}_\nu\mathbf{R}_\alpha\mathbf{R}_\beta\mathbf{R}_\gamma\}\,. \tag{16}$$

For the field ansatz (3) it may be reduced to the remarkably elegant expression:

$$S_{\text{WZ}} = \frac{\mathrm{i}N_c}{2\pi^2}\int \mathrm{d}^4x\, \frac{1}{4!}\,\mathrm{Tr}\left\{\left(\overline{\mathbf{p}} + \overline{\mathbf{p}'}\right)^3\overline{\mathbf{k}}\right\}\,. \tag{17}$$

Here we have used differential 1-form notations:

$$\overline{\mathbf{p}} = \left(\mathbf{d}\sqrt{\mathbf{U}_\pi}\right)\sqrt{\mathbf{U}_\pi}^{\dagger},$$

$$\overline{\mathbf{p}'} = \sqrt{\mathbf{U}_\pi}^{\dagger}\left(\mathbf{d}\sqrt{\mathbf{U}_\pi}\right),$$

$$\overline{\mathbf{k}} = (\mathbf{d}\mathbf{U}_K)\,\mathbf{U}_K^{\dagger}\,. \tag{18}$$

Explicit evaluation of (17) upon division by the normalization factor (12) yields

$$L_{\text{WZ}} = -\mathrm{i}\frac{C_3(\lambda,\mu)}{C_2(\lambda,\mu)}\frac{3N_c}{80f_K^2}\frac{1}{2\pi^2} \times \int F'\sin^2 F\cdot\left(k^\dagger\dot k - \dot k^\dagger k\right)\,\mathrm{d}r\,. \tag{19}$$

In the case of the fundamental SU(3) group representation $(1,0)$ this result agrees with that in Ref. [6] up to the overall factor 1/16, which derives from the different notation of f_K and the present choice of SU(3) group generators.

2.4. The symmetry breaking term

The SU(3) chiral symmetry breaking term of Lagrangian density is defined as [10]

$$\mathcal{L}_{\text{SB}} = \frac{f_\pi^2}{4}\left[m_0^2\,\mathrm{Tr}\left\{\mathbf{U} + \mathbf{U}^\dagger - 2\cdot\mathbf{1}\right\} - 2m_8^2\,\mathrm{Tr}\left\{\hat J^{(1,1)}_{(0,0,0)}\left(\mathbf{U} + \mathbf{U}^\dagger\right)\right\}\right]. \tag{20}$$

Here (with the exception of the case of the self-adjoint representation) model parameters

$$m_0^2 = \frac{1}{3}\left(m_\pi^2 + 2\frac{f_K^2}{f_\pi^2}m_K^2\right), \tag{21}$$

$$m_8^2 = \frac{10}{3\sqrt{3}}\frac{C_2(\lambda,\mu)}{C_3(\lambda,\mu)}\left(m_\pi^2 - \frac{f_K^2}{f_\pi^2}m_K^2\right) \tag{22}$$

are related to pion and kaon masses m_π and m_K respectively.

For the self-adjoint representations $\lambda = \mu$ the symmetry breaking term is proportional only to m_0^2 because the trace of the second term is equal to zero.

Substitution of the ansatz (3) into (20) leads to the symmetry breaking term for any irreducible SU(3) group rep. The explicit form of the general symmetry breaking term is

$$\mathcal{L}_{\mathrm{SB}(\lambda,\mu)} = -\mathcal{M}_{\mathrm{SB}(\lambda,\mu)} = \frac{f_\pi^2}{2}\left(m_0^2 Q_{11}^{\lambda,\mu} - \frac{2}{\sqrt{3}} m_8^2 Q_{12}^{\lambda,\mu}\right) - \frac{f_\pi^2}{4f_K^2}\left(m_0^2 Q_{21}^{\lambda,\mu} - \frac{2}{\sqrt{3}} m_8^2 Q_{22}^{\lambda,\mu}\right) K^\dagger K\,. \quad (23)$$

Here the notation is

$$Q_{11}^{\lambda,\mu} = \sum_{z,j,m}^{\lambda,\mu} \cos[2mF(r)] - \dim(\lambda,\mu)\,,$$

$$Q_{12}^{\lambda,\mu} = \sum_{z,j}^{\lambda,\mu} \left\{[(\lambda-\mu)+3z] \sum_{m=-j}^{j} \cos[2mF(r)]\right\},$$

$$Q_{21}^{\lambda,\mu} = \sum_{z,j,m}^{\lambda,\mu} \left\{A_{z,j,m}^{\lambda,\mu} \cos[2mF(r)]\right\},$$

$$Q_{22}^{\lambda,\mu} = \sum_{z,j,m}^{\lambda,\mu} \left\{[(\lambda-\mu)+3z] A_{z,j,m}^{\lambda,\mu} \cos[2mF(r)]\right\}, \quad (24)$$

and

$$A_{z,j,m}^{\lambda,\mu} = -\frac{1}{2j(j+1)} \times \Big\{2j^3 + j^4 - mz\,(1+z+\lambda)\,(z-\mu-1) - j\left[\lambda - 3z^2 - 2z\lambda + m(3z+\lambda-\mu) + \mu + 2z\mu + \lambda\mu\right] - j^2\left[\lambda - 3z^2 - 2z\lambda - 1 + m(3z+\lambda+\mu) + \mu + 2z\mu + \lambda\mu\right]\Big\}. \quad (25)$$

For the fundamental SU(3) representation $(1,0)$ the symmetry breaking term reduces to the simple form:

$$L_{\mathrm{SB}(1,0)} = -8\pi \int r^2 f_\pi^2 m_\pi^2 \sin^2\frac{F}{2}\,\mathrm{d}r - \frac{1}{4f_K^2}\int r^2 \left(f_K^2 m_K^2 - f_\pi^2 m_\pi^2 \sin^2\frac{F}{2}\right) k^\dagger k\,\mathrm{d}r\,. \quad (26)$$

For higher representations the weight of symmetry breaking grows, especially the heavy meson mass part. This term is crucial for phenomenologically realistic spectra of the heavy flavour hyperons.

3. Canonical quantization of the soliton

For the quantization of the Skyrme soliton for arbitrary reducible SU(3) group representations (λ,μ) the quantum operators and soliton field may be separated in the usual way as

$$\mathbf{U}(\mathbf{r},\mathbf{q}(t)) = \mathbf{A}(\mathbf{q}(t))\,\mathbf{U}_\pi(\mathbf{r})\,\mathbf{A}^\dagger(\mathbf{q}(t))\,. \quad (27)$$

The operator $\mathbf{A}$ may be expressed as a direct sum of Wigner D matrices:

$$\mathbf{A}(\mathbf{q}(t)) = \sum_{z,j}^{(\lambda,\mu)} \oplus D^j(\mathbf{q})\,. \quad (28)$$

The three real time dependent parameters $\mathbf{q}(t) = \{q_1(t), q_2(t), q_3(t)\}$ are quantum variables, which represent Euler angles of rotation of the soliton. Considering the Skyrme Lagrangian quantum mechanically *ab initio*, the generalized coordinates $\mathbf{q}(t)$ and velocities $\mathbf{q}(t)$ have to satisfy the commutation relations [11]:

$$\left[\dot{q}^a, q^b\right] = -f^{ab}(\mathbf{q})\,. \quad (29)$$

Here the tensor $f^{ab}(\mathbf{q})$ is a function of generalized coordinates $\mathbf{q}$ only. It is symmetric as a consequence of the commutation relation $\left[q^a, q^b\right] = 0$.

The explicit form of $f^{ab}(\mathbf{q})$ can be determined only after imposition of the quantization conditions. Using Weyl ordering of the operators, the commutation relation between a generalized velocity component $\dot{q}^a$ and an arbitrary function $G(\mathbf{q})$ is given by

$$\left[\dot{q}^a, G(\mathbf{q})\right] = -\mathrm{i} f^{ab}(\mathbf{q}) \frac{\partial}{\partial q^b} G(\mathbf{q})\,. \quad (30)$$

After substituting (27) into Skyrme soliton Lagrangian (2), the dependence on generalized velocities can be expressed as

$$L(\mathbf{q},\mathbf{q},F) = \frac{1}{2} a(F)\dot{q}^a g_{ab}(\mathbf{q})\dot{q}^b + \mathcal{O}\left(\mathbf{q}^0\right). \quad (31)$$

The function g_{ab} is interpreted as a metric tensor which can be expressed as a product of functions $C_a^{\prime(0,1,m)}(\mathbf{q})$ [11]:

$$g_{ab}(\mathbf{q}) = -\frac{1}{4}\dim(\lambda,\mu) C_2(\lambda,\mu)\,(-1)^m \times C_a^{\prime(0,1,m)}(\mathbf{q})\, C_b^{\prime(0,1,-m)}(\mathbf{q})\,. \quad (32)$$

Here the soliton moment of inertia $a(F)$ is defined as

$$a(F) = \frac{8\pi}{3e^3 f_\pi}\int \tilde{r}^2 \sin^2\tilde{F}\left(1 + \tilde{F}'^2 + \frac{1}{\tilde{r}^2}\sin^2\tilde{F}\right)\mathrm{d}\tilde{r}\,. \quad (33)$$

The canonical momentum p_a conjugate to the generalized coordinate q^a is

$$p_a = \frac{1}{2}a(F)\left\{\dot{q}^b, g_{ba}(\mathbf{q})\right\}. \tag{34}$$

Employment of the canonical commutation relations leads to the explicit form of the tensor $f^{ab}(\mathbf{q})$:

$$f^{ab}(\mathbf{q}) = [a(F)g_{ab}(\mathbf{q})]^{-1}. \tag{35}$$

Following [11] we define the angular momentum operator

$$I_a = -\mathrm{i}\left\{p^b, C'^{(0,1,a)}_{-b}(\mathbf{q})\right\}$$
$$= (-1)^a\,\frac{\mathrm{i}a(F)}{2}\left\{\dot{q}^b, C'^{(0,1,a)}_{-b}(\mathbf{q})\right\}, \tag{36}$$

which may be recognized as iso-rotation operator of the soliton.

The explicit form of canonically quantized soliton after division by the factor (12) becomes

$$\hat{L} = -M_{\mathrm{cl}}(F) - \Delta M_{(\lambda,\mu)}(F) + \frac{\hat{\mathbf{I}}^2}{2a(F)} + L_{\mathrm{SB}(\lambda,\mu)}\,. \tag{37}$$

Here $\Delta M_{(\lambda,\mu)}(F)$ is quantum mass correction to the classical soliton mass:

$$\Delta M_{(\lambda,\mu)}(F) = -\frac{2\pi}{5a^2(F)}\frac{1}{e^3 f_\pi}$$
$$\times\int\tilde{r}^2\Big\{5-11\tilde{F}'^2-\sin^2\tilde{F}\left(16-16\tilde{F}'^2+\frac{3}{2\tilde{r}^2}\right)$$
$$+\,3C_2(\lambda,\mu)\left[4\sin\tilde{F}\left(1-\tilde{F}'^2\right)+4\tilde{F}'^2\right.$$
$$\left.+\,\frac{1}{\tilde{r}^2}\sin^2\tilde{F}\right]\Big\}\sin^2\tilde{F}\,\mathrm{d}\tilde{r}\,. \tag{38}$$

The corresponding Hamilton operator for the quantum soliton finally is

$$\hat{H}_{(\lambda,\mu)}(F) = M_{\mathrm{cl}}(F) + \Delta M_{(\lambda,\mu)}(F) + \frac{\hat{\mathbf{I}}^2}{2a(F)}$$
$$+\,M_{\mathrm{SB}(\lambda,\mu)}(F)\,. \tag{39}$$

The energy of the canonically quantized soliton differs from the semiclassically quantized soliton by the appearance of the mass correction $\Delta M_{(\lambda,\mu)}$, which depends on representation. This mass correction is negative and lowers energy of quantum soliton. It shows that quantum soliton is only approximately "rigid body".

Minimization of the energy of quantum soliton leads to an integro-differential equation for the quantum chiral angle $F(\tilde{r})$. The boundary conditions are the same as for classical chiral angle: $F(0) = \pi$, $F(\infty) = 0$. At large distances ($\tilde{r} \to \infty$), quantum chiral angle equation takes the asymptotic form:

$$\tilde{r}^2\tilde{F}'' + 2\tilde{r}\tilde{F}' - (2 + \tilde{m}^2_{\mathrm{eff}}\tilde{r}^2)\tilde{F} = 0\,, \tag{40}$$

here the quantity $\tilde{m}^2_{\mathrm{eff}}$ is defined as

$$\tilde{m}^2_{\mathrm{eff}} = \tilde{m}^2_\pi - \frac{e^4}{3\tilde{a}(F)}\left(8\Delta M_{(\lambda,\mu)} + \frac{2i(i+1)+3}{\tilde{a}(F)}\right). \tag{41}$$

Here we have used the notation $\tilde{m} = (ef_\pi)^{-1}m$ and $i(i+1)$ is the eigenvalue of the operator $\hat{\mathbf{I}}^2$. The solution of the asymptotic equation is

$$F(\tilde{r}) = C\left(\frac{\tilde{m}^2_{\mathrm{eff}}}{\tilde{r}} + \frac{1}{\tilde{r}^2}\right)\exp(-\tilde{m}^2_{\mathrm{eff}}\tilde{r})\,, \tag{42}$$

where C is an arbitrary constant determined numerically by tuning the asymptotic solution to the numerical solution.

The requirement for the quantum soliton to be stable with finite mass sets the restriction $\tilde{m}^2_{\mathrm{eff}} > 0$. The stability of the quantum soliton is ensured by the term (38). Its absence in the semiclassical approach leads to the instability of the solution [13]. Furthermore, the appearance of the quantum mass part breaks the scale invariance of the equation of motion, which is the symmetry of the classical Lagrangian. This shows that Skyrme Lagrangian has an anomaly. Therefore, the positive parameter $m_{\mathrm{eff}} = ef_\pi\,\tilde{m}_{\mathrm{eff}}$ can be interpreted as an effective pion mass.

3.1. Semiclassical approach to the heavy flavour meson field

In the semiclassical approach the heavy flavour meson field is described in the "rest frame" of the background soliton. In this slowly corotating meson–soliton system the meson effectively becomes an object with isospin zero and spin effectively equal to the quantum number u [5]. Thus a kaon bound in the $u = \frac{1}{2}$ wave can be effectively treated as a strange quark. The same treatment is approximately valid for bound charm and bottom mesons and leads to bound states describing charm and bottom baryons.

The semiclassical part of the bound state Lagrangian after substituting (3) into (2) for the $u=\frac{1}{2}$ wave meson and after dividing by the normalization factor (12) is

$$\delta L = -\frac{1}{4f_K^2}\frac{1}{2}\delta a(F,k)\dot{\alpha}^2 + \frac{1}{4f_K^2}\dot{\alpha}_0 t_0 \int \tilde{r}^2 \mathrm{d}\tilde{r}\, \left[\mathrm{i}\,\chi\left(\tilde{k}^\dagger\dot{\tilde{k}} - \dot{\tilde{k}}^\dagger\tilde{k}\right)\right]. \quad (43)$$

Here

$$\dot{\alpha}_m = \mathrm{i}\frac{1}{2}\left\{\dot{q}^\alpha, C_\alpha^{\prime(0,1,m)}(\mathbf{q})\right\},$$

$$\delta a(F,k) = \frac{2}{e^3 f_\pi} \times\int\Bigg\{\left[\left(\frac{1}{4}-\frac{1}{3}\sin^2\tilde{F}\right)+\frac{1}{16}\left(\tilde{F}'^2+\frac{2}{\tilde{r}^2}\sin^2\tilde{F}\right)\right]\tilde{k}^2 - \sin^2 F\left(\frac{3}{8}\tilde{F}'^2\sin^2\tilde{F}+\frac{1}{3\tilde{r}^2}\sin^2\tilde{F}\right)\tilde{k}^2 - \frac{1}{8}\frac{\mathrm{d}}{\mathrm{d}\tilde{r}}\big(\tilde{F}'\sin 2\tilde{F}\big)\tilde{k}^2 + \frac{1}{6}\sin^2\tilde{F} \times\left(\tilde{k}'^2+\frac{2l(l+1)}{\tilde{r}^2}\tilde{k}^2\right)\Bigg)\Bigg\}\tilde{r}^2\mathrm{d}\tilde{r},$$

$$\chi = \frac{1}{e^2}\left[\frac{1}{4}\frac{(-1)^l}{2l+1}\left(\tilde{F}'^2\cos\tilde{F}+\frac{2}{\tilde{r}^2}\sin^2\tilde{F}\right) - \frac{1}{4\tilde{r}^2}\left(\frac{3(-1)^l}{2l+1}-1\right)\frac{\mathrm{d}}{\mathrm{d}\tilde{r}}\big(\tilde{r}^2\tilde{F}'\sin\tilde{F}\big) + \frac{1}{6}\tilde{F}'^2\sin^2\frac{\tilde{F}}{2} - \frac{l(l+1)}{3\tilde{r}^2}\sin^2\tilde{F} + \frac{2}{3}\sin^2\frac{\tilde{F}}{2}+\frac{(-1)^l}{2l+1}\cos\tilde{F}\right]. \quad (44)$$

Here $\delta a(F,k)$ is a small positive parameter, which can be interpreted as an additional contribution to the soliton moment of inertia. This shows that the background field rotating together with quantum soliton slows it down somewhat. The operators multiplied with $\chi(\tilde{r})$ are responsible for the spin–spin interaction of soliton and the bound meson. These operators allow discriminating between states with different total spin.

On the semiclassical level the Wess–Zumino action

$$S_{\mathrm{WZ}} = -\frac{\mathrm{i}N_c}{2\pi^2}\int_M \frac{1}{4!}\mathrm{Tr}\left\{\overline{\mathbf{w}}^3\left(\overline{\alpha}' + W^\dagger\overline{\alpha}'W\right)\right\} \quad (45)$$

contributes to the spin–spin interaction between the soliton and the heavy flavour meson. Here $\overline{\mathbf{w}} = \mathbf{d}UU^\dagger$ and $\overline{\alpha}' = \alpha'^\dagger\mathbf{d}\alpha'$ are 1-forms. The semiclassical part of Wess–Zumino Lagrangian can be written in the compact form

$$\delta L_{\mathrm{WZ}} = \dot{\alpha}_0 t_0 \frac{1}{4f_K^2}\int \tilde{r}^2\mathrm{d}\tilde{r}\,\chi_{\mathrm{wz}}(\tilde{r},\lambda,\mu)\,\tilde{k}^2, \quad (46)$$

where

$$\chi_{\mathrm{wz}}(\tilde{r},\lambda,\mu) = \frac{C_3(\lambda,\mu)}{C_2(\lambda,\mu)}\frac{1}{10\pi^2}\frac{\tilde{F}'}{\tilde{r}^2}\Bigg\{\sin^2\frac{\tilde{F}}{2} \times\Big[3+5\cos\tilde{F}+4\cos 2\tilde{F}-2\big(\cos\tilde{F}-2\cos 2\tilde{F}\big)\Big] - l(l+1)\Big[2\big(\cos\tilde{F}+\cos 2\tilde{F}\big)-\cos 3\tilde{F}-4\Big] + l(l+1)\frac{1}{2}\big(\cos\tilde{F}+4\cos 2\tilde{F}-3\cos 3\tilde{F}\big)\Bigg\}. \quad (47)$$

3.2. The bound state mechanics

The final expression of the semiclassical Lagrangian is

$$L = -M_{\mathrm{cl}} - \Delta M_{(\lambda,\mu)} + \frac{1}{2}[a(F)+\delta a(F,k)]\dot{\alpha}^2 + \frac{f_\pi}{e}\int\tilde{r}^2\mathrm{d}\tilde{r}\,\Big[f\,\dot{\tilde{k}}^2 + \mathrm{i}\Lambda(\tilde{r},\lambda,\mu)\left(\tilde{k}^\dagger\dot{\tilde{k}}-\dot{\tilde{k}}^\dagger\tilde{k}\right) - h\,\tilde{k}'^2 - \big(M_K^2(\tilde{r},\lambda,\mu)+V_{\mathrm{eff}}\big)\tilde{k}^2\Big] - M_{\mathrm{SB}(\lambda,\mu)} + \dot{\alpha}_0 u_0\int\tilde{r}^2\mathrm{d}\tilde{r}\,\Big[\mathrm{i}\chi\left(\tilde{k}^\dagger\dot{\tilde{k}}-\dot{\tilde{k}}^\dagger\tilde{k}\right)+\chi_{\mathrm{wz}}(\tilde{r},\lambda,\mu)\tilde{k}^2\Big]. \quad (48)$$

Here the coefficients coming from the Wess–Zumino and symmetry breaking terms depend on the chosen representation:

$$\Lambda(\tilde{r},\lambda,\mu) = -\frac{e^2}{4}\frac{3}{5}\frac{C_3(\lambda,\mu)}{C_2(\lambda,\mu)}\frac{N_c}{2\pi^2\tilde{r}^2}\tilde{F}'\sin^2\tilde{F}, \quad (49)$$

$$M_\pi^2(\tilde{r},\lambda,\mu) = -\frac{1}{2}\tilde{m}_0^2 Q_{11}^{\lambda,\mu} + \frac{1}{\sqrt{3}}\tilde{m}_8^2 Q_{12}^{\lambda,\mu}, \quad (50)$$

$$M_K^2(\tilde{r},\lambda,\mu) = \tilde{m}_0^2 Q_{21}^{\lambda,\mu} - \frac{2}{\sqrt{3}}\tilde{m}_8^2 Q_{22}^{\lambda,\mu}. \quad (51)$$

In the case of the fundamental SU(3) group representation these take the simple forms

$$\Lambda\left(\tilde{r},1,0\right)=-\frac{e^2N_c}{8\pi^2\tilde{r}^2}\sin^2\tilde{F}\cdot\tilde{F}'\,,$$

$$M_\pi^2\left(\tilde{r},1,0\right)=2\tilde{m}_\pi^2\;\sin^2\frac{\tilde{F}}{2}\,,$$

$$M_K^2\left(\tilde{r},1,0\right)=-\tilde{m}_\pi^2\sin^2\frac{\tilde{F}}{2}+\frac{f_K^2}{f_\pi^2}\tilde{m}_K^2\,.$$

The equation of motion for the meson field corresponding to Lagrangian (48) is

$$\begin{aligned}0=&-f\,\ddot{\tilde{k}}+2\mathrm{i}\left[\Lambda\left(\tilde{r},\lambda,\mu\right)+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right]\dot{\tilde{k}}\\&-\left[V_{\text{eff}}+M_K^2\left(\tilde{r},\lambda,\mu\right)-\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi_{\text{wz}}\left(\tilde{r},\lambda,\mu\right)\right]\tilde{k}\\&+\frac{1}{\tilde{r}^2}\left(\tilde{r}^2\,h\,\tilde{k}'\right)'\,.\end{aligned}\qquad(52)$$

Here a term $\delta a(F,k)$ has been dropped, as it is of order N_c^{-2}, which is not relevant here.

After substitution of (9) we get two independent equations that represent states with strangeness $S=-1$ and $S=1$ respectively:

$$\begin{aligned}&\left[\tilde{\omega}_n^2f+2\tilde{\omega}_n\left(\Lambda+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right)-V_{\text{eff}}-M_K^2\right.\\&\left.+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi_{\text{wz}}\right]\tilde{k}_n+\frac{1}{\tilde{r}^2}\left(\tilde{r}^2\,h\,\tilde{k}_n'\right)'=0\,,\end{aligned}\qquad(53)$$

$$\begin{aligned}&\left[\tilde{\omega}_{-n}^2f-2\tilde{\omega}_{-n}\left(\Lambda+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right)-V_{\text{eff}}-M_K^2\right.\\&\left.+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi_{\text{wz}}\right]\tilde{k}_{-n}+\frac{1}{\tilde{r}^2}\left(\tilde{r}^2\,h\,\tilde{k}_{-n}'\right)'=0\,.\end{aligned}\qquad(54)$$

Here $\tilde{\omega}=(ef_\pi)^{-1}\omega$.

Upon diagonalization of the Hamiltonian we find that the canonical momentum conjugate to $\tilde{k}$ is

$$\tilde{\pi}_m=f\,\dot{\tilde{k}}_m^\dagger+\mathrm{i}\left(\Lambda+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right)\tilde{k}_m^\dagger\,.\qquad(55)$$

The canonical commutation relations lead to the following orthogonality relations:

$$\begin{aligned}&\frac{1}{4f_K^2}\int\mathrm{d}\tilde{r}\,\tilde{r}^2\tilde{k}_n\tilde{k}_m\left[(\tilde{\omega}_n+\tilde{\omega}_m)f\right.\\&\left.+2\left(\Lambda+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right)\right]=\delta_{nm}\,,\end{aligned}\qquad(56)$$

$$\begin{aligned}&\frac{1}{4f_K^2}\int\mathrm{d}\tilde{r}\,\tilde{r}^2\tilde{k}_{-n}\tilde{k}_{-m}\left[(\tilde{\omega}_{-n}+\tilde{\omega}_{-m})f\right.\\&\left.-2\left(\Lambda+\frac{e}{f_\pi}\dot{\alpha}_0u_0\chi\right)\right]=\delta_{nm}\,.\end{aligned}\qquad(57)$$

In terms of creation and annihilation operators which obey the usual algebra

$$\left[a_n,a_m^\dagger\right]=\delta_{nm}\,,\qquad\left[b_n,b_m^\dagger\right]=\delta_{nm}\,,\qquad(58)$$

the diagonalized Hamilton operator for kaon fields becomes

$$\hat{H}=\tilde{\omega}_na_n^\dagger a_n+\tilde{\omega}_{-n}b_{-n}^\dagger b_{-n}\,.\qquad(59)$$

The momentum canonically conjugate to the quantum degrees of freedom is

$$\begin{aligned}p_\alpha'=&-a'(F,k)\frac{1}{2}\left\{\dot{q}^\beta,g_{\beta\alpha}\left(\mathbf{q}\right)\right\}\\&+\mathrm{i}C_\alpha'^{(0,1,0)}\left(\mathbf{q}\right)u_0\chi'(F,k,\lambda,\mu)\,,\end{aligned}\qquad(60)$$

where

$$a'(F,k)=a(F)+\delta a(F,k)\,,\qquad(61)$$

$$\begin{aligned}&\chi'(F,k,\lambda,\mu)=\\&\int\tilde{r}^2\mathrm{d}\tilde{r}\left[\mathrm{i}\chi\left(\tilde{k}^\dagger\dot{\tilde{k}}-\dot{\tilde{k}}^\dagger\tilde{k}\right)+\chi_{\text{wz}}\left(\tilde{r},\lambda,\mu\right)\tilde{k}^2\right].\end{aligned}\qquad(62)$$

This differs from (34) because of the influence of the bound meson field. Now we can write the final expression of the angular momentum operator of soliton rotating with the bound field:

$$I_a'=(-1)^aa'(F,k)\dot{\alpha}_a+\delta_{a,0}\,t_0\,\chi'(F,k,\lambda,\mu)\,.\qquad(63)$$

It differs from (36) by additional part which arises because of the interaction with bound field. Using the canonical Legendre transformation

$$H=\frac{1}{2}\left\{\dot{q}^\alpha,p_\alpha\right\}+\int\tilde{r}^2\mathrm{d}\tilde{r}\,\left(\tilde{\pi}\dot{\tilde{k}}+\dot{\tilde{k}}^\dagger\tilde{\pi}^\dagger\right)-L\,,\qquad(64)$$

Table 1. Hyperon mass spectra with $l = 1$, MeV.

	$\Lambda\,(1116)$			$\Sigma\,(1193)$			$\Sigma^*\,(1385)$		
irrep. \ m_π	0	71.6	137	0	71.6	137	0	71.6	137
$(1,0)$	1049	1029	990	–	1235	1201	–	1355	1374
$(2,0)$	1015	1067	1036	1221	1205	1178	1330	1320	1305
$(2,1)$	1082	1196	1159	1330	1310	1272	1425	1411	1383

the following Hamiltonian is obtained:

$$\hat{H} = M_{\text{cl}} + \Delta M_{(\lambda,\mu)} + M_{\text{SB}(\lambda,\mu)} + \omega + \frac{1}{2a'(F,k)}\left[\hat{\mathbf{l}}'^2 - u_0^2\chi'^2(F,k,\lambda,\mu)\right] . \quad (65)$$

4. Interpretation of physical states and numerical results

Quantum states of soliton are identified as isospin states. Total angular momentum j of soliton is equal to its isospin, $(i,j) = (i,i)$. Lowest energy state of bound field has $l = 1$ [5]. The spin–spin interaction terms let one to distinguish states with different total spin. Thus we recognize the final state with $(0,1/2) = (0,0) + (0,1/2)$ to be $\Lambda\,(P_{01})$, state with $(1,1/2) = (1,1) + (0,1/2)$ to be $\Sigma\,(P_{11})$, and state with $(1,3/2) = (1,1) + (0,1/2)$ to be $\Sigma^*\,(P_{13})$.

Bound state model has 5 independent parameters – model parameter e, pion decay constant f_π, pion mass m_π, kaon decay constant f_K, and kaon mass m_K for calculating properties of hyperons. For calculations of properties of charmed or bottom baryons correspondingly we have to use decay constant f_D or f_B and meson mass m_D or m_B. All these parameters are measured experimentally except the model parameter e, the value of which can be calculated by setting quantum soliton with isospin $i = \frac{1}{2}$ mass equal to the experimentally measured mass of the nucleon,

$$M_N = \frac{f_\pi}{e} M_{\text{cl}}(\tilde{F}) + e^3 f_\pi \left[\Delta M_{(\lambda,\mu)}(\tilde{F}) + \frac{3}{8a(\tilde{F})}\right] + f_\pi^2 M_{SB(\lambda,\mu)}(\tilde{F}) . \quad (66)$$

We have calculated hyperon mass spectra for wave $l = 1$ using three different values of pion mass parameter m_π in the symmetry breaking term (20). We have chosen input parameters to be $M_N = 939$ MeV, $f_\pi = 65.35$ MeV, $f_K/f_\pi = 1.22$, $m_K = 495$ MeV and three different choices of the pion mass parameter in the symmetry breaking term: $m_\pi = 0$ MeV, $m_\pi = 71.6$ MeV, and $m_\pi = 137$ MeV. The results for irreps $(1,0)$, $(2,0)$, and $(2,1)$ are presented in Table 1 (the dash means that we have found no stable solutions). The value $m_\pi = 0$ means that pions at classical level are treated as pure Goldstone bosons but they acquire mass at quantum level. The value $m_\pi = 137$ MeV represents the classical mass of the pion in the symmetry breaking term. Although, by setting $m_\pi = 0$, from (41) we get $m_{\text{eff}} = 108$ MeV and by setting $m_\pi = 137$ we get $m_{\text{eff}} = 193$ MeV. By fitting calculations to the nucleon properties we have found the value of the symmetry breaking parameter to be $m_\pi = 71.6$ MeV, which leads to the correct effective mass of the pion $m_{\text{eff}} = 137$ MeV. Consequent calculation results for charmed and bottom baryons for irrep $(1,0)$ are given in Tables 2 and 3. The input parameters for charmed baryons are $f_D/f_\pi = 1.7$, $m_K = 1867$ MeV and for bottom baryons $f_B/f_\pi = 2$, $m_K = 5279$ MeV.

Table 2. Charmed baryons spectra for irrep. $(1,0)$ with $l = 1$, MeV.

	$\Lambda_c\,(2287)$			$\Sigma_c\,(2455)$			$\Sigma_c^*\,(2520)$		
m_π	0	71.6	137	0	71.6	137	0	71.6	137
	2198	2155	2068	–	2492	2355	–	2658	2606

Table 3. Bottom baryons spectra for irrep. $(1,0)$ with $l = 1$, MeV.

	$\Lambda_b\,(5620)$			$\Sigma_b\,(5810)$			$\Sigma_b^*\,(5830)$		
m_π	0	71.6	137	0	71.6	137	0	71.6	137
	5584	5485	5284	–	6236	5865	–	6325	6177

Table 4. Calculated f_π, MeV.

irrep. \ m_π	0	116	137
$(1,0)$	58	54.4	53
$(2,0)$	60	57.6	56.8
$(2,1)$	61.8	60	58.5

It was shown in Ref. [13] that there is an alternative way to do the calculations. One has to choose isoscalar part of nucleon electric mean square radius

$$\left\langle r^2_{E,I=0}\right\rangle = -\frac{2}{\pi}\frac{1}{(ef_\pi)^2}\int \tilde{r}^2 \mathrm{d}\tilde{r}\,\tilde{F}'\sin^2\tilde{F} \qquad (67)$$

as input parameter instead of pion decay constant. Now f_π and e are model parameters and can be calculated from equations (66) and (67), however, the ratio f_K/f_π (f_D/f_π or f_B/f_π) is kept fixed as previously. We choose input parameter to be $\left\langle r^2_{E,I=0}\right\rangle = 0.604$ fm^2. By fitting calculations to the nucleon properties we find the value of the symmetry breaking parameter to be $m_\pi = 116$ MeV. Calculated values of parameter f_π for some irreps and m_π values are given in Table 4. Hyperon calculation results for wave $l = 1$ are given in Table 5 and results for wave $l = 0$ are given in Table 6. Calculations of charmed and bottom baryons are given in Tables 7 and 8.

5. Discussion

In this paper, we have discussed the bound state model describing heavy baryons containing a single heavy quark. We have constructed a bound state out of canonically quantized soliton and heavy meson. Soliton was quantized canonically in the framework of the collective coordinates formalism for arbitrary irreducible SU(3) representation. We have treated soliton field quantum mechanically *ab initio*. The canonical quantization of the soliton respecting noncommutativity of quantum variables – collective coordinates, which are the Euler angles of the soliton rotation – leads to quantum soliton stabilizing term. This term depends on the representation (λ,μ) and lowers soliton mass. Bound meson field was treated semiclassically. The symmetry breaking and Wess–Zumino terms play a crucial role for the bound field and also depend on the representation. For self adjoint representation $\lambda = \mu$ the Wess–Zumino term vanishes and symmetry breaking term is restricted to SU(2) symmetry breaking term. The bound state approach was done precisely respecting canonical Lagrangian and Hamiltonian formalism.

We found a semiclassical Hamiltonian describing bound states in the background of the quantum soliton. The representation (λ,μ) influences the explicit expression of Hamiltonian and tunes effective Yukawa potential. Consequently, the dependence on representation can be interpreted as a new discrete phenomenological parameter of the model. However, the explicit physical meaning of the dependence on representation is not completely understood.

The calculations were done for the spectra of the strange, charm, and bottom baryons, where they were treated as bound states of a quantum soliton and an appropriate flavour meson. The predicted mass values for the non-excited hyperons are very close to the experimental ones. Although, the canonical approach in not very successful in describing excited states. The same remarks are valid for charm and bottom flavoured baryons. However, we were able to investigate charmed and bottom baryons only in the fundamental SU(3) rep. Also we put a lower bound on the ratio $f_B/f_\pi \geq 2$. The energies of calculations for higher reps are far too high because of rapidly growing influence of symmetry breaking term. Nevertheless, the results could drasti-

Table 5. Hyperon mass spectra with $l = 1$, MeV.

	Λ (1116)			Σ (1193)			Σ^* (1385)		
irrep. \ m_π	0	116	137	0	116	137	0	116	137
$(1,0)$	1128	1098	1086	–	1202	1190	–	1324	1318
$(2,0)$	1118	1089	1078	1231	1191	1180	1319	1287	1278
$(2,1)$	1223	1186	1174	1327	1280	1266	1414	1375	1364

Table 6. Hyperon mass spectra with $l = 0$, MeV.

	Λ (1405)			Σ (1660)			Σ^* (1670)		
irrep. \ m_π	0	116	137	0	116	137	0	116	137
$(1,0)$	–	1288	1278	–	1479	–	–	1418	–
$(2,0)$	1249	–	–	–	–	–	–	–	–
$(2,1)$	1307	–	–	–	1446	–	–	1355	–

Table 7. Charmed baryons spectra for rep. $(1,0)$, MeV.

	Λ_c (2287)			Σ_c (2455)			Σ_c^* (2520)		
$l \setminus m_\pi$	0	116	137	0	116	137	0	116	137
$l=1$	2343	2251	2219	–	2374	2328	–	2563	2531
	Λ_c (2593)			Σ_c (2800)					
$l=0$	–	2468	2435	–	–	–			

Table 8. Bottom baryons spectra for rep. $(1,0)$, MeV.

	Λ_b (5620)			Σ_b (5810)			Σ_b^* (5830)		
$l \setminus m_\pi$	0	116	137	0	116	137	0	116	137
$l=1$	5824	5533	5436	–	5797	5659	–	6020	5900
	Λ_b (?)								
$l=0$	–	5770	5673						

cally change if different mass term were employed. The mass term is very important because the heavy meson is treated semiclassicaly. Therefore, right mass term could lead to a complete set of states of charmed and bottom baryons.

Appendix

Elements of SU(3) group algebra

The SU(3) group generators are defined as components of irreducible $(1,1)$ tensors. Their relation to the Gell-Mann generators Λ_k are:

$$\begin{aligned}
J^{(1,1)}_{(0,0,0)} &= -\tfrac{1}{2}\Lambda_8 \,, \\
J^{(1,1)}_{(0,1,0)} &= \tfrac{1}{2}\Lambda_3 \,, \\
J^{(1,1)}_{(0,1,1)} &= -\tfrac{1}{2\sqrt{2}}\left(\Lambda_1 + \mathrm{i}\Lambda_2\right) \,, \\
J^{(1,1)}_{(0,1,-1)} &= \tfrac{1}{2\sqrt{2}}\left(\Lambda_1 - \mathrm{i}\Lambda_2\right) \,, \\
J^{(1,1)}_{(-\frac{1}{2},\frac{1}{2},\frac{1}{2})} &= \tfrac{1}{2\sqrt{2}}\left(\Lambda_4 + \mathrm{i}\Lambda_5\right) \,, \\
J^{(1,1)}_{(\frac{1}{2},\frac{1}{2},-\frac{1}{2})} &= \tfrac{1}{2\sqrt{2}}\left(\Lambda_4 - \mathrm{i}\Lambda_5\right) \,, \\
J^{(1,1)}_{(-\frac{1}{2},\frac{1}{2},-\frac{1}{2})} &= \tfrac{1}{2\sqrt{2}}\left(\Lambda_6 + \mathrm{i}\Lambda_7\right) \,, \\
J^{(1,1)}_{(\frac{1}{2},\frac{1}{2},\frac{1}{2})} &= -\tfrac{1}{2\sqrt{2}}\left(\Lambda_6 - \mathrm{i}\Lambda_7\right) \,.
\end{aligned} \tag{A1}$$

In the case of the fundamental representation Λ_k the matrices reduce to the standard Gell-Mann matrices λ_k.

The generators $J^{(1,1)}_{(Z,I,M)}$ obey the hermitean conjugation relation:

$$\left(J^{(1,1)}_{(Z,I,M)}\right)^\dagger = (-1)^{Z+M} J^{(1,1)}_{(-Z,I,-M)} \,. \tag{A2}$$

The action of the operators $J^{(1,1)}_{(Z,I,M)}$ on the basis states and the commutation relations are given in [10]. The dimension of an arbitrary representation (λ,μ) is denoted by

$$\dim(\lambda,\mu) = \frac{1}{2}\left(\lambda+1\right)\left(\mu+1\right)\left(\lambda+\mu+2\right) \,. \tag{A3}$$

The explicit expressions for the quadratic and cubic Casimir operators are, respectively,

$$\begin{aligned}
C_2(\lambda,\mu) &= \tfrac{1}{3}\left(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu\right) \,, \\
C_3(\lambda,\mu) &= \tfrac{1}{162}\left(\lambda-\mu\right)\left(\lambda+2\mu+3\right)\left(2\lambda+\mu+3\right) \,.
\end{aligned} \tag{A4}$$

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SUNKIEJI BARIONAI KAIP KANONIŠKAI KVANTUOTI SKYRME'OS MODELIO SOLITONAI SURIŠTŲJŲ BŪSENŲ ARTINYJE

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Santrauka

Sunkieji barionai, turintys vieną keistąjį, žavingajį ar gelminį kvarką, tirti Callano ir Klebanovo pasiūlytame Skyrme'os modelio surištųjų būsenų artinyje. Topologinis solitonas yra kanoniškai kvantuojamas bet kuriame SU(3) grupės įvaizdyje, o kvantuojant atsiradęs neigiamas masės dėmuo stabilizuoja kvantinį solitoną. Šiame artinyje sunkiųjų aromatų mezonai solitono lauke traktuojami pusiauklasiškai, todėl tiems laisvės laipsniams užrašoma ir išsprendžiama surištųjų būsenų lygtis. Wesso ir Zumino narys yra labai svarbus modelio lagranžiane. Be jo surištosios būsenos neegzistuoja, o hiperonų masių spektre šis narys išskiria teigiamo ir neigiamo keistumo būsenas. Apskaičiuotas keistųjų, žavingųjų ir gelminių sunkiųjų barionų masių spektras ir nustatyta jo priklausomybė nuo grupės įvaizdžio. Skaičiavimų rezultatai palyginti su eksperimentiniais duomenimis.