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THE DUAL PROPERTY OF NUMBER AND VELOCITY FLUCTUATIONS OF CHARGE CARRIERS IN A MACROSCOPIC CONDUCTOR UNDER THERMODYNAMIC EQUILIBRIUM CONDITIONS

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Fluctuation-dissipation relations are complemented by relating the macrovariables conductance and resistance, that describe dissipation, to the microvariables variance of carrier number and drift velocity fluctuations, that are the noise sources for constant voltage and constant current operation conditions, respectively. Thermal equilibrium implies a relationship between these two noise sources which follows from the reciprocity property of conductance and resistance. The boundary conditions of the measurement select the proper microscopic source of fluctuations to be related to the dissipation. An important consequence is that the source of shot noise, being associated with fluctuations of the carrier number inside the sample, is already present under equilibrium conditions, while the time scale of the source changes from an effective transport time to a current transit time when going from equilibrium to nonequilibrium conditions.

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1. Introduction

It is a common interpretation that thermal noise of a macroscopic conductor [1–3] is associated with velocity fluctuations due to the Brownian motion of free carriers inside the sample. By contrast, in the presence of an imposed current, for independent carriers the shot noise associated with fluctuations of the number of carriers due to their granular nature is evidenced in the standard form $S_I = 2qI$ [4], with S_I being the low-frequency current spectral-density, and q being the absolute value of the charge responsible of the steady current I. In the past years the presence of shot noise emerged as an ubiquitous phenomenon in mesoscopic conductors [5]. As a consequence, the concept of the existence of two independent kinds of noise, thermal, synonymous of equilibrium conditions, and shot, synonymous of nonequilibrium conditions, has taken place [5, 6].

The aim of this paper is to go beyond this concept of the two noise sources by proving that thermal noise can be also interpreted as originating from carrier number fluctuations. Indeed, the space and time symmetry of the thermal equilibrium conditions allows for a dual representation of thermal noise. The two representations are complementary with respect to each other according to the chosen boundary conditions taken to detect the fluctuating macrovariable, i. e. current or voltage. By analogy with the standard expressions of the fluctuations of the macroscopic variables (i. e. the spectral densities of current and voltage fluctuations) in terms of the dissipation (i. e. conductance and resistance) we provide the expressions of dissipation in terms of the microscopic source of fluctuations (i. e. carrier number or drift-velocity). In this way, the theoretical frame of the fluctuation-dissipation relations in the limit of zero frequency are fully assessed through their dual representation, i. e. the dissipation-fluctuation relations. We provide a formulation that is independent of the kind of statistics, thus applying to classical as well as to fermionic and bosonic particles. A series of interesting consequences will be further presented and discussed.

2. Theory

Current and voltage fluctuations of a two-terminal sample at thermodynamic equilibrium are described by the Callen–Welton fluctuation-dissipation theorem

[3] that in the limit of zero frequency coincides with the Nyquist relations [2]

$$S_I = 4\frac{\overline{I^2}}{\Delta f} = 4k_{\mathrm{B}}TG\,, \quad (1)$$

$$S_V = 4\frac{\overline{V^2}}{\Delta f} = 4k_{\mathrm{B}}TR\,, \quad (2)$$

with S_I and S_V being the spectral densities of instantaneous current and voltage fluctuations, respectively (we recall that being at equilibrium their average values are identically zero). $\overline{I}^2$ and $\overline{V}^2$ are the variances of current and voltage fluctuations, respectively, Δf is the bandwidth of interest (microscopically related to the temporal profile of the corresponding correlation function), k_{B} is the Boltzmann constant, T is the absolute temperature, G is the conductance, and R is the resistance of the two-terminal sample. Here and henceforth, the bar over physical quantities denotes the ensemble average.

From a phenomenological microscopic approach, that uses the generalized Einstein relation and Ohm's law, it is

$$\overline{I^2} = \frac{q^2\,\overline{\delta N^2}}{\tau_N^2}\,, \quad (3)$$

$$\overline{V^2} = L^2\,\overline{\delta E^2} = \frac{L^2\,\overline{\delta v_{\mathrm{d}}^2}}{\mu^2}\,, \quad (4)$$

with $\overline{\delta N^2}$ being the variance of the total number of carriers inside the sample, τ_N a characteristic time that will be detailed later, L the length of the sample, $\overline{\delta E^2}$ the variance of electric-field fluctuations averaged over the sample length, $\overline{\delta v_{\mathrm{d}}^2}$ the variance of carrier drift-velocity fluctuations while moving inside the sample, and μ being the carrier mobility.

The choice of the boundary conditions is of crucial importance since the fluctuating current is that one measured in the outside short circuit (constant-voltage conditions) while the fluctuating voltage is that one measured at the terminals in the outside open circuit (constant-current conditions). Here the electrical contacts are assumed to be perfectly ohmic and of negligible length. Furthermore, the contacts do not play any role for the kind of statistics obeyed by carriers.

From a macroscopic point of view, the dual property of the fluctuation-dissipation theorem is a consequence of the reciprocal property of the linear-response coefficients (like Norton's and Thévenin's theorems in electrotechnics),

$$GR = 1. \quad (5)$$

From a microscopic point of view, this dual property is more intriguing since current fluctuations are associated with fluctuations of the total number of carriers instantaneously present in the short-circuited sample, while voltage fluctuations are associated with the instantaneous steady-state (drift) velocity of carriers inside the open-circuited sample. Indeed, by using the generalized Einstein relation [7], holding both for classical and quantum statistics (in the case of bosons here and in the following we consider only temperatures above the critical temperature for the Bose–Einstein condensation [8]), we have

$$G = \left(\frac{q}{L}\right)^2 D\frac{\partial \overline{N}}{\partial \mu_0}\,, \quad (6)$$

with μ_0 being the chemical potential, and

$$D = \overline{v^2}\tau \quad (7)$$

is the analogously generalized diffusion coefficient [7], with the quadratic velocity averaged over the differential distribution function with respect to the carrier number (the so-called differential quadratic velocity) given by

$$\overline{v^2} = \frac{\sum_{\mathbf{k}} v^2 \frac{\partial f(\epsilon_{\mathbf{k}})}{\partial N}}{\sum_{\mathbf{k}} \frac{\partial f(\epsilon_{\mathbf{k}})}{\partial N}}\,. \quad (8)$$

Here, $\mathbf{k}$ is the carrier wave vector, $\epsilon_{\mathbf{k}}$ is the corresponding energy, f is the distribution function, and $\tau = 1/\Delta f$ is a characteristic time describing transport properties like diffusion and mobility, which therefore also describes the bandwidth in Eq. (1).

Using the general relation for number fluctuations in the grand canonical ensemble (which is again independent of the type of statistics [9])

$$\overline{\delta N^2} = k_{\mathrm{B}}T\frac{\partial \overline{N}}{\partial \mu_0}, \quad (9)$$

the conductance can be written as

$$G = \left(\frac{q}{L}\right)^2 \frac{D}{k_{\mathrm{B}}T}\overline{\delta N^2}. \quad (10)$$

Accordingly, for the spectral density of current fluctuations at zero frequency we have

$$S_I = 4\left(\frac{q}{L}\right)^2 D\,\overline{\delta N^2}. \quad (11)$$

The expressions given above correlate the conductance to the fluctuations of the total carrier number through the diffusion coefficient and, in particular, using Eq. (3) they imply

$$\tau_N^2 = \frac{L^2}{\overline{v^2}}. \tag{12}$$

Thus, τ_N can be interpreted as an effective transport time through the sample [10]. Analogously, from the voltage fluctuations and the Nyquist relation (Eqs. (2) and (4)) we obtain

$$S_V = 4\frac{L^2}{\mu^2 \Delta f}\,\overline{\delta v_\mathrm{d}^2} = 4\frac{L^2 m^2}{q^2 \tau}\,\overline{\delta v_\mathrm{d}^2}, \tag{13}$$

where we have used again $\Delta f = 1/\tau$ as well as the relation

$$\mu = \frac{q\tau}{m} \tag{14}$$

for the mobility with the carrier effective mass m. We thus find an equivalent relation, announced here for the first time, that correlates the resistance with the variance of the instantaneous carrier drift velocity fluctuations:

$$R = \frac{L^2 m^2}{q^2 \tau k_\mathrm{B} T}\overline{\delta v_\mathrm{d}^2}. \tag{15}$$

The reciprocity property in Eq. (5) implies the dual property inter-relating fluctuations of the total carrier number with those of the drift velocity in the respective operation conditions

$$\overline{\delta N^2}\ \overline{v^2} = \overline{N}^2\ \overline{\delta v_\mathrm{d}^2}, \tag{16}$$

where we have used

$$\overline{\delta v_\mathrm{d}^2} = \frac{k_\mathrm{B} T}{m\overline{N}}, \tag{17}$$

which follows from the general relation for the variance of the drift velocity fluctuations

$$\overline{\delta v_\mathrm{d}^2} = \frac{k_\mathrm{B} T}{\overline{N}^2}\sum_{\mathbf{k}} v_x^2 \frac{\partial f(\epsilon_{\mathbf{k}})}{\partial \mu_0}. \tag{18}$$

The relation (16) together with the definitions coming from statistics and given in Eqs. (8), (9) and (17) is the main result of the paper. The relation (16) is formally independent of the type of statistics [9], even if the values of the average quantities depend on the type of statistics.

In other words, at thermodynamic equilibrium the carrier total-number fluctuations in the constant-voltage scheme are inter-related to carrier drift velocity fluctuations in the constant-current scheme. Notice that all the above expressions hold for classical as well as for Fermi or Bose degenerate statistics, thus complementing the fluctuation-dissipation theorem in the limit of zero frequency.

3. Conclusions

The paper reports the dual property of the fluctuation-dissipation theorems by relating the dissipation to the microscopic noise sources, what we call the dissipation-fluctuation relations. These relations are given in a form that is independent of the type of statistics, thus including classical, Fermi–Dirac and Bose–Einstein statistics (the latter above the critical temperature for Bose–Einstein condensation). From a physical point of view, the temperature entering the Nyquist relations (1) and (2) is here interpreted in dynamical terms and associated with the variance of instantaneous carrier total-number or drift velocity fluctuations, as dictated by statistics. For this purpose, we go beyond the classical interpretation, generally accepted in the literature, that relates mobility (i. e. dissipation) to the fluctuations of the single carrier velocity [11].

The main results can be summarized as follows. Equations (10) and (15) should be called dissipation-fluctuations relations, since they relate the dissipative quantities, conductance and resistance, directly with the associated macroscopic fluctuations expressed in terms of their microscopic sources.

By expressing current fluctuations in terms of the ratio between the variance of carrier number fluctuations and the effective transport time τ_N we show that thermal noise can also be associated with fluctuations of carrier number, that is with the discreteness of the electrical charge. Furthermore, for fermions the vanishing of the low-frequency current spectral-density at $T = 0$ is associated with the vanishing of the variance of carrier number fluctuations, i. e. with the instantaneous correlation (coherence) between appearance and disappearance of an elemental carrier number fluctuation inside the sample. This is essential because here diffusion differs from zero even at zero temperature, and thus the notion of diffusion, as synonymous of noise, fails completely. Accordingly, the source of shot noise is already present even at thermal equilibrium, as described by the generalized Schottky formula [4, 12]

$$S_I = 2qI \coth\left(\frac{qV}{2k_\mathrm{B}T}\right). \tag{19}$$

Indeed, the Schottky formula above merely represents the transition from thermal equilibrium to nonequilibrium conditions when a steady current is imposed by an applied voltage.

Sometimes it is erroneously reported that shot noise adds to thermal noise, which leads to an artificial over-estimate of the noise. From a physical point of view, the source of shot noise is the same as that of thermal noise under constant voltage condition, i. e. fluctuations of the total number of carriers inside the sample. However, at thermal equilibrium the time scale of current fluctuations is related to the effective transport time as $\tau_N = L/\sqrt{\overline{v^2}}$, while in the high voltage limit, $V \gg k_B T/q$, the time scale is associated with the current transit time, i. e. $\tau_T = L/v_d$. In both cases this time scale is associated with the transit time spent by carrier number fluctuation in going from one contact to the opposite one, and the difference by a factor 2 in the high voltage limit is related to a symmetry breaking since the current selects only the path from an injector to a collector to dissipate a carrier number fluctuation outside the sample [13]. One should notice that the generalized Schottky formula above holds only for independent or distinguishable carriers. Interaction among carriers or their indistinguishability is responsible under far from equilibrium conditions of shot-noise suppression (carrier antibunching because of a repulsive character of the interaction) or enhancement (carrier bunching because of an attractive character of the interaction). In contrast, for the classical case it is the absence of motion at $T = 0$, i. e. $D = 0$, that is responsible for the vanishing of current noise. For the case of voltage fluctuations, the same limit at $T = 0$ is associated with the tendency of the drift velocity fluctuations to approach zero which is again a property which is independent of the kind of statistics.

We have thus shown that the reciprocal property given in Eq. (16) interrelates the current and voltage fluctuations in close analogy with the reciprocity of macroscopic conductance and resistance.

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KRŪVININKŲ SKAIČIAUS IR GREIČIO FLIUKTUACIJŲ DUALUMAS MAKROSKOPINIAME LAIDININKE TERMODINAMINĖS PUSIAUSVYROS SĄLYGOMIS

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