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Influence of supply voltage and frequency variations on the electrical equipment and power consumption in LV and MV distribution networks

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This paper is focused on the analysis of supply voltage and frequency quality in low voltage and medium voltage distribution power systems. The influence of supply voltage and frequency variations, within the limits defined in the valid standards, on the power network's parameters has been evaluated using the stochastic theory and the statistical analysis methods. The circuit of RLC-load was considered for numerical evaluation. The probability distribution of supply voltage and frequency was defined and the suggestions on optimum voltage and frequency quality parameters regarding electric energy consumption, power losses and electrical equipment lifespan were given.

Keywords: voltage variation, frequency variation, power quality, electric energy meter, normal distribution, Pearson's chi-squared test, power distribution system

INTRODUCTION

The power system's management should be aimed at maintaining such work parameters that the required reliability and power quality (PQ) are ensured. This means that one should not, although it is technically possible, strive to maintain absolute reliability and PQ, 'driving' the parameters of the power system mode into the required framework, since it is not cost-effective. Though, it is crucial that PQ is constantly maintained at the proper level.

The principles of power system's operation and control are based on certain power quality requirements, which are formulated in domestic and international standards. For most standardized PQ indicators normally acceptable and maximum permissible values are established. Herewith, the values should not go beyond the maximum permissible values and should be within the range of normally acceptable values with 0.95 probability, while the measurement time interval is not less than 24 h. These requirements must be respected in all normal, repair and post-emergency conditions, except for those caused by natural disasters and unforeseen circumstances (e.g. hurricane, earthquake, flood, fire, etc.).

Power quality is characterized by the voltage quality and the frequency quality of the alternating current (AC) voltage. The voltage quality is evaluated by several indicators, most of which are characterized by acceptable values. Among others, the following important parameters are listed and defined in the EN 50160-2010:

- nominal voltage,
- frequency of the supply voltage,
- long interruption of the supply voltage,
- short interruption of the supply voltage,
- rapid voltage change,
- voltage dip,
- voltage fluctuation,
- voltage swell,
- voltage unbalance,
- voltage variation,
- flicker severity,
- harmonic voltage.

EN 50160 gives the main voltage parameters and their permissible deviation ranges at the customer's point of common coupling in public low voltage (LV), medium voltage (MV) and high voltage (HV) power systems, under normal operating conditions.

One of the most important PQ indicators is the root mean square (RMS) value of the voltage magnitude variation at a given point in the network, which is understood as slow smooth deviations from the nominal or desired value that occur continuously over time. Quantitatively, the voltage variations are estimated by the value of the steady-state voltage deviation

$$\Delta U = U - U_{rated}, \tag{1}$$

$$\delta U = \frac{U - U_{rated}}{U_{rated}} 100\%, \tag{2}$$

where U_{rated} is the rated operating voltage and U is the actual voltage value.

Power systems' voltage stability depends on the relationships between the load bus voltage magnitude, the reactive power injection or absorption and the power supplied to the load [1]. Thus, voltage variations are mainly due to load pattern, changes of load or nonlinear loads, and, as a result, changes of the voltage drop in the network's elements. Another reason for the voltage deviations is a change in the voltage profile in the power centers, i.e. on the buses of power plants or secondary voltage buses of step-down substations, where distribution power systems are connected [2].

Earlier in Ukraine, the state standard GOST 13109-07 was in force, where the normally acceptable and maximum permissible values of voltage magnitude variation were set as $\delta U_{norm} = \pm 5\%$ and $\delta U_{max} = \pm 10\%$, respectively. Since 2014, power quality in Ukrainian LV and MV power networks has been standardized in accordance with EN 50160-2010, which is being used in the countries of the European Union (EU). EN 50160-2010 indicates that the range of variation of the root mean square (rms) magnitude of supply voltage is ±10% for 95% of the 10-minute mean RMS values over each one-week period.

The frequency of supply voltage is the frequency of fundamental harmonic voltage, measured as the mean value during a given time interval. In Ukraine and in the EU, the rated frequency is 50 Hz. To assess the quality of the frequency the parameter of frequency deviation is used, which is understood as slow smooth changes of frequency (less than 1% per second) relative to its nominal value

$$\delta f = \frac{f - f_{rated}}{f_{rated}} 100\%, \tag{3}$$

where f_{rated} is the rated frequency and f is the actual frequency value.

The reason for frequency deviation is the imbalance of the generated and consumed active power in the electric power system. The former state standard GOST 13109-07 sets the normally acceptable and maximum permissible values of frequency variation as $\delta f_{norm} = \pm 0.2$ Hz and $\delta f_{max} = \pm 0.4$ Hz, respectively. As specified in EN 50160-2010, the network frequency has to be

- for interconnected supply systems: 50 Hz ± 1% (49.5–50.5 Hz) for 95% and 50 Hz – 6%...+4% (47–52 Hz) for 100% of a week;
- for non-interconnected supply systems: 50 Hz ± 2% for 95% and 50 Hz ± 15% for 100% of a week.

For networks operating in an islanded condition the frequency is allowed to vary within a wider range.

The requirements of EN 50160-2010 regarding voltage and frequency deviations are less strict, compared to those of GOST 13109-97. It should be noted that EN 50160 is principally informative and accepts no responsibility when the limits are exceeded. On the other hand, the consumers' point of view is usually totally different – they regard the limits given in EN 50160 as requirements that must be guaranteed by the supplier. However, for some consumers even fulfilling the requirements given in EN 50160 does not assure a satisfactory level of PQ. In such cases, the level of PQ required must be defined in a separate agreement between a utility and a consumer.

The objective of this study is to estimate the current situation about supply voltage and frequency quality and to evaluate how their deviations within the thresholds defined in the valid standards affect the parameters of an electric network (e.g. power and currents at the consumer's side, load power factor), by means of the stochastic theory and the statistical analysis methods. On the other hand, this study aims to define the probability distribution of supply voltage and frequency, and to find the optimum voltage quality parameters regarding power consumption and losses.

CAUSES OF VOLTAGE AND FREQUENCY VARIATIONS AND THE WAYS TO MAINTAIN VOLTAGE AND FREQUENCY STABILITY

Undesired impacts of voltage variations and voltage control strategies

Supply voltage quality is a complex term concerning voltage deviations from its ideal characteristics. A practical approach for the analysis of supply voltage quality parameters by means of stochastic theory is discussed in [3]. The main problem for consumers is often supposed to be too low voltage in the feeder. As the network's impedance varies slightly, the voltage variation is mainly due to load current variations. Therefore, the voltage on the transformer's secondary side is often stepped up to ensure the rated voltage level during peak loads [3]. The other issue influencing PQ is the rising penetration level of distributed generation (DG) in LV distribution networks. The paper [4] attempts to quantify the possible negative impacts on the voltage profile of different deployment levels of DG in secondary networked distribution systems. The authors determined the critical amount of distributed energy resources (DERs) that the LV networks can withstand without exhibiting undervoltage and overvoltage problems or unexpected load tripping. DG can provide voltage support, maintaining its appropriate value at the end of the feeder. However, the voltage may exceed the upper standardized limit in the points of DG injections. The paper [5] assesses the impact of high photovoltaic (PV) penetration on voltage profiles in residential neighbourhoods. The simulation results show that the PV penetration level does not adversely influence the voltage profile of a typical distribution grid when the DERs capacity does not exceed 2.5 kW per household on average [5]. However, power generation by PV-panels and their connection to the network through current converters can negatively affect PQ [6]. The resonance phenomenon in the LV networks with a high number of distributed power inverters for PV-panels and the recommendations to improve the PQ characteristics of the inverters are discussed in [7].

Voltage variations affect the operation of both power consumers and the electrical network. For example, the rotation speed of the rotor of an asynchronous motor changes with voltage deviations. If the voltage on the stator's winding drops by 15%, then the torque on the shaft will decrease by 1/4, and the asynchronous motor will most likely stop or, if it comes to starting, will not start at all. With a reduced supply voltage, the current consumption increases, the stator's windings heat up more, and the service life of the motor will be greatly reduced. If the motor constantly operates at a supply voltage of 90% of the rated value, then its service life will be reduced by half. If the supply voltage exceeds the rated value by 1%, then the reactive power consumption of a motor will increase by about 5%, and the overall efficiency of such a motor will decrease.

Negative voltage deviations cause decrease in illumination, which may influence the labour productivity in enterprises that require eye strain. With an increase in voltage magnitude by 10%, the service time of incandescent lamps decreases 4 times, and with a voltage decrease by 10%, the light flux of an incandescent lamp decreases by 40%, while the flux of fluorescent lamps decreases by 15%. If the voltage turns out to be 90% of the rated value when the fluorescent lamp is turned on, then it will flicker, and at 80% it will not start at all. Voltage variations affect power losses in transformers and power lines [2].

Also some loads may drop off by themselves without any action of protective relays when voltage is unsustainable. Table 1 summarizes the examples of interruption voltages due to distribution protection [8].

Table 1. **Voltage variation tolerances for load-and-control equipment**

Device	Voltage deviation
Communication equipment	±5%
Computers, data processing equipment	±10%
Contactors, motor starts	–15%...+10%
Illumination – Fluorescent – Incandescent	 –10%, –25% +18%
Induction motors	±10%
Resistive loads, furnaces, heaters	variable

There are two ways to meet the requirements of EN 50160 regarding the voltage magnitude variations. The first is to reduce power losses, the second is to regulate the voltage. The voltage drop in the electrical distribution network can be defined as

$$\Delta U = \frac{P \cdot R + Q \cdot X}{U_{sc(subst)}}, \tag{4}$$

where P and Q are the active and the reactive power transmitted through the line, respectively, R and X are resistance and the reactance of the line, respectively, and $U_{sc(subst)}$ is the voltage at the power supply center or at the transformer substation.

Ways to reduce power losses are the following:

- R optimization: the cross-section of the line conductors should be chosen in accordance with the minimum possible loss conditions. The economic feasibility of using a conductor with a larger cross-sectional area should be proven vs having larger power losses but with a thinner (and cheaper) line conductor.
- X optimization: longitudinal compensation of line reactance which, however, enhances the danger of high short-circuit currents if $X \to 0$.
- Q reduction: installation of static VAr compensators to reduce reactive power transmission through the distribution power system. The desired effect can be achieved by means of capacitor banks or synchronous motors operating under over-excitation. Additionally, compensation of Q allows one to reduce power losses and save electric energy, since the overall power losses will decrease.
- Adequate choice of power transformers for the expected power flow (i.e. satisfying an optimal transformer load factor).

Transformers in the supply center regulate the voltage U_{sc}. They are equipped with automatic devices for adjusting the transformation coefficient in accordance with the current loading. Regulation under load is possible in a range of ±16...20% of the rated voltage. Transformers at intermediate substations U_{subst} can perform voltage regulation as well. The windings of these transformers are equipped with switchable taps, which are designed for the ±5% control range with a step of 2.5%. Switching is made without excitation when the transformer is disconnected from the grid.

R and X are to be chosen at the designing stage of the power network, and further operational change of these parameters is impossible. Q and U_{subst} can be adjusted during network loading seasonal changes, but it is necessary to manage the operating modes of reactive power compensation centrally, in accordance with the current operating mode of the network as a whole, which is the responsibility of an electric energy supplying utility. As for the voltage regulation U_{sc} from the power center, this is the most convenient way for the supplying utility, which allows one to adjust voltage properly, according to the load schedule of the networks.

Under operating conditions, it is not possible to constantly monitor voltage deviations at each electric device. Therefore, in the power transmission and distribution systems the so-called control points are set, where voltage tolerance thresholds are assessed. If the voltage in these most characteristic points is within acceptable limits, then it does not go beyond the acceptable limits for the most consumers as well. Test points are usually selected on the secondary voltage buses at the main load nodes, as well as on the buses of power plants.

Influence of frequency variation on the electrical equipment

Frequency variation directly affects the operation of power consumers. For instance, the frequency deviations worsen the operation of electric motors, causing changes in their rotation frequency, active and reactive power consumption. The highest danger of frequency deviation is for the equipment of power plants. The performance of mechanisms that overcome the static pressure (e.g. feed pumps for the power plant's auxiliaries, which overcome the high pressure from the boiler) is reduced [2]. In addition, the reduced frequency in the power network affects the service life of steel-containing equipment (e.g. electric motors, transformers), due to an increase in the magnetization current and additional heating of steel elements. However, frequency variation has an insignificant effect on the operation of furnaces and lighting loads.

For the power system the dependences of active P and reactive Q power on frequency is shown in Fig. 1 [2].

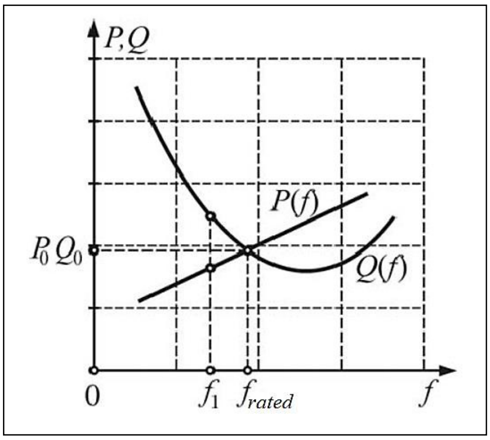


Fig. 1. Static frequency load characteristics

The dependencies in Fig. 1 are static frequency load characteristics. When system frequency decreases due to the active power generation deficiency, consumers reduce their load, trying to maintain frequency at the appropriate level. As can be seen from Fig. 1, the frequency decrease to f_1 leads to an increase in the reactive power Q consumed by the load, which entails a decrease in voltage at the point of load connection. In this case, the active power P consumption is reduced. Typically, the increase in reactive power consumption is higher than the decrease in active power, which leads to an increase in total power flows over the network elements and, consequently, to higher power and energy losses.

At any time in the power system a balance of active power is maintained:

$$\Sigma P_G = \Sigma P_L - \Sigma \Delta P. \tag{5}$$

Here ΣP_G is the total power of generators at power plants, ΣP_L is the total power consumption of the power system, including in-house loads of power plants, and $\Sigma \Delta P$ is the total power losses in electrical networks.

At any frequency the power generated by power plants is equal to power consumption. At the same time, the nominal value of frequency in the power system indicates that the generated power is sufficient to cover the normal needs of consumers. A lower than rated frequency indicates a shortage of generated power, and a higher frequency indicates an excess of power in the system.

To control the frequency, the turbines of generators are equipped with speed controllers. At the event of a power shortage and a sharp frequency decrease as a result of primary and secondary regulation, all power plants will become fully loaded. If the power of generators disconnected during an accident is greater than the reserve at all power plants of the system, then the frequency will not be restored to its rated value. During the event of a significant power shortage the frequency decrease will be high. To prevent an avalanche of frequency, automatic quick-action measures must be taken. Frequency recovery is carried out by automatic frequency unloading, at which some consumers are disconnected [2]. Deliberate curtailment of some consumers allows one to save generating capacities in operation, to provide power supply for the rest of the loads.

TESTING THE HYPOTHESIS ABOUT THE DISTRIBUTION LAW OF THE RANDOM VARIABLES OF VOLTAGE AND FREQUENCY

According to the data provided by the local electricity distributor for the feeder, which supplies household loads (single-story residential buildings) in a rural area, the results of voltage (Table 2) and frequency (Table 3) measurements were

Table 2. Observations of random voltage value

No.	Deviation, ±%	Overvoltage deviations regarding U_{rat}			Undervoltage deviations regarding U_{rat}		
		Number of results, *n*	U/U_{rat}	*U* random value, V	Number of results, *n*	U/U_{rat}	*U* random value, V
1	0	264	1	220	191	0.995	218.9
2	0.5	201	1.005	221.1	179	0.99	217.8
3	1	181	1.01	222.2	176	0.985	216.7
4	1.5	158	1.015	223.3	174	0.98	215.6
5	2	154	1.02	224.4	162	0.975	214.5
6	2.5	152	1.025	225.5	150	0.97	213.4
7	3	150	1.03	226.6	144	0.965	212.3
8	3.5	144	1.035	227.7	140	0.96	211.2
9	4	140	1.04	228.8	132	0.955	210.1
10	4.5	134	1.045	229.9	118	0.95	209
11	5	118	1.05	231	94	0.945	207.9
12	5.5	92	1.055	232.1	76	0.94	206.8
13	6	72	1.06	233.2	64	0.935	205.7
14	6.5	62	1.065	234.3	50	0.93	204.6
15	7	48	1.07	235.4	38	0.925	203.5
16	7.5	45	1.075	236.5	24	0.92	202.4
17	8	39	1.08	237.6	16	0.915	201.3
18	8.5	23	1.085	238.7	22	0.91	200.2
19	9	12	1.09	239.8	15	0.905	199.1
20	9.5	10	1.095	240.9	15	0.9	198
21	10	10	1.1	242	8	0.895	196.9
22	10.5	8	1.105	243.1	6	0.89	195.8
23	11	7	1.11	244.2	6	0.885	194.7
24	11.5	6	1.115	245.3	6	0.88	193.6
25	12	6	1.12	246.4	6	0.875	192.5
26	12.5	3	1.125	247.5	3	0.87	191.4
27	13	1	1.13	248.6	3	0.865	190.3
28	13.5	1	1.135	249.7	2	0.86	189.2
29	14	1	1.14	250.8	1	0.855	188.1
30	14.5	1	1.145	251.9	1	0.85	187
31	15	1	1.15	253	–	–	–

compiled. The results are shown for phase A. Measurements were taken at the feeder endpoint, i.e. for the last consumer on the line. A total of 4 266 measurements were made for the voltage value and 1 987 measurements for the frequency value. The measurements were taken at 1 min step.

According to the data from Table 2, the dependence of *f*(*U*), Fig. 2, was built.

According to the data from Table 3, the dependence of *f*(*f*), Fig. 3, is built.

Estimating the shape of the voltage and frequency bar charts, it was hypothesized that the dependences *f*(*U*) and *f*(*f*) obey the normal (Gaussian) distribution, and the probability density function of the variable under study coincides with the Gaussian function [9]

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \cdot e^{-\frac{(x_i - m_x)^2}{2\cdot\sigma_x^2}}, \qquad (6)$$

Table 3. **Observations of random frequency value**

No.	Deviation, ±%	Overvoltage deviations regarding f_{rat}			Undervoltage deviations regarding f_{rat}		
		Number of results, *n*	f/f_{rat}	*f* random value, Hz	Number of results, *n*	f/f_{rat}	*f* random value, Hz
1	0	142	1	50	99	0.9996	49.98
2	0.04	70	1.0004	50.02	98	0.9992	49.96
3	0.08	68	1.0008	50.04	87	0.9988	49.94
4	0.12	67	1.0012	50.06	76	0.9984	49.92
5	0.16	66	1.0016	50.08	65	0.998	49.9
6	0.2	66	1.002	50.1	66	0.9976	49.88
7	0.24	64	1.0024	50.12	64	0.9972	49.86
8	0.28	63	1.0028	50.14	62	0.9968	49.84
9	0.32	62	1.0032	50.16	60	0.9964	49.82
10	0.36	60	1.0036	50.18	55	0.996	49.8
11	0.4	56	1.004	50.2	52	0.9956	49.78
12	0.44	45	1.0044	50.22	44	0.9952	49.76
13	0.48	26	1.0048	50.24	38	0.9948	49.74
14	0.52	22	1.0052	50.26	34	0.9944	49.72
15	0.56	20	1.0056	50.28	24	0.994	49.7
16	0.6	18	1.006	50.3	18	0.9936	49.68
17	0.64	16	1.0064	50.32	11	0.9932	49.66
18	0.68	11	1.0068	50.34	8	0.9928	49.64
19	0.72	9	1.0072	50.36	6	0.9924	49.62
20	0.76	6	1.0076	50.38	6	0.992	49.6
21	0.8	6	1.008	50.4	7	0.9916	49.58
22	0.84	4	1.0084	50.42	4	0.9912	49.56
23	0.88	4	1.0088	50.44	4	0.9908	49.54
24	0.92	4	1.0092	50.46	3	0.9904	49.52
25	0.96	3	1.0096	50.48	2	0.99	49.5
26	1	2	1.01	50.5	3	0.9896	49.48
27	1.04	2	1.0104	50.52	3	0.9892	49.46
28	1.08	1	1.0108	50.54	0	0.9888	49.44
29	1.12	1	1.0112	50.56	1	0.9884	49.42
30	1.16	1	1.0116	50.58	1	0.988	49.4
31	1.2	1	1.012	50.6	–	–	–

where m_x is the mean or expectation of the distribution (and also its median and mode), σ_x is the standard deviation, and x_i is the random value of the investigated variable.

If the values of voltage and frequency obey the Gaussian distribution, the results of the experiments should be processed using the following numerical characteristics of the random variables and their systems [9]:

1. Mean or expectation (the mean probability of a random variable)

$$M[x] = m_x = \int_{-\infty}^{\infty} xf(x)dx\,. \qquad (7)$$

Mathematical assessment of the mean of expectation

$$M^*[X] = m^*_x = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad (8)$$

where n is the number of tests taken (sample's volume).

2. Variance of a random variable (a measure of the variation of a given random variable, i.e. its deviation from the mathematical expectation)

$$D[x] = D_x = \int_{-\infty}^{\infty} (x - m_x)^2 f(x)dx. \qquad (9)$$

Mathematical assessment of the variance

$$D^*[X] = D^*_x = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - m^*_x)^2, \qquad (10)$$

or

$$D^*[X] = D^*_x = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right). \qquad (11)$$

3. Standard deviation (the scatterplot of the values of a random variable relative to its median)

$$\sigma_x = \sqrt{D_x}\,. \qquad (12)$$

Mathematical assessment of the standard deviation

$$\sigma^*_x = \sqrt{D^*_x}\,. \qquad (13)$$

Voltage and frequency variations in the electrical network have a stochastic nature. Means of expectation for both values can be obtained from the data given in Tables 2 and 3, using the formulas (7) and (8). Get $m_U = 220.526$ V and $m_f = 49.9998$ Hz.

The Pearson's chi-squared test [10] was employed to validate the hypothesis of a normal distribution of random voltage U and frequency f. At $n = 4266$, the voltage measurement results concentrated in the range 187 V $\le U_{rated} \le$ 253 V. This range was divided into 12 intervals of 5.5 V width, according to the Sturges' rule [11]

$$l = 1 + [3.322 \cdot lgn], \qquad (14)$$

where l is the number of intervals.

The frequency measurement results were concentrated in the range 49.4 Hz $\le f_{rated} \le$ 50.6 Hz. According to (14), this range was divided into 12 intervals of 0.1 Hz width.

The experimental value of the chosen criterion can be found by

$$\chi^2_{\text{exp}} = \sum_{i=1}^{l} \frac{(\gamma_i - np_i)^2}{np_i}, \qquad (15)$$

where γ_i is the empirically obtained investigated value, p_i is the probability of a random variable X falling into the i-th interval, and h is the width of a single interval.

Assuming the significance level $\alpha = 0.05$ and the number of degrees of freedom $k_U = 9$ for voltage and $k_f = 9$ for frequency, the critical values of the Pearson's chi-squared test were determined: $\chi^2_{\text{cr}(U)} = 16.919$ and $\chi^2_{\text{cr}(f)} = 16.919$. Here the degree of freedom of the random value x is defined as

$$k_x = l - 1 - r, \qquad (16)$$

where r is the number of parameters of a supposed distribution law (if a distribution law is expected to be normal, then the mean or expectation and the standard deviation are assessed, $r = 2$).

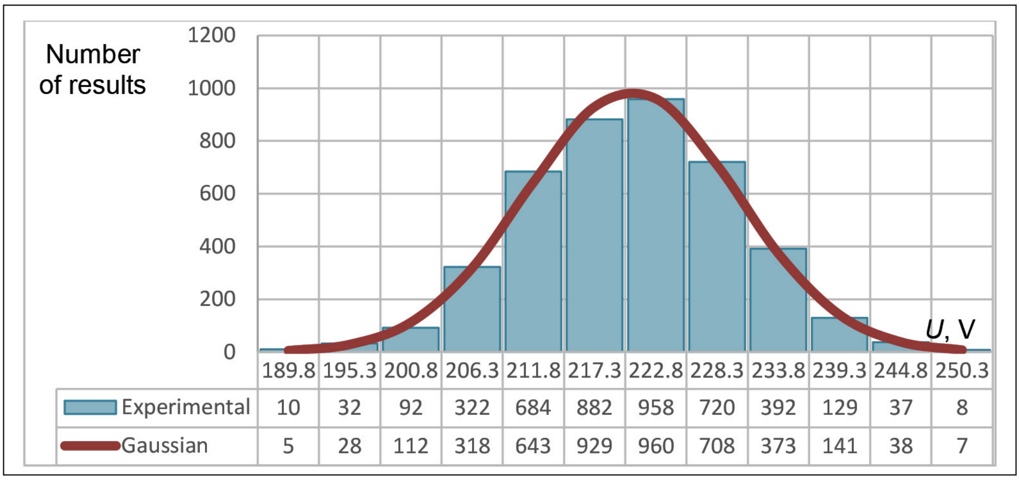
After the calculation, the experimental values of the Pearson's chi-squared test were obtained: $\chi^2_{\text{exp}(U)} = 16.058$ and $\chi^2_{\text{exp}(f)} = 16.581$. In both cases, the condition $\chi^2_{\text{exp}(x)} < \chi^2_{\text{cr}(x)}$ is satisfied, therefore, the null hypothesis of the normal (Gaussian) distribution of random voltage and frequency in the considered power grid is not rejected.

The agreement between the empirical and theoretical normal distribution is statistically significant. Using the 3σ rule, a threshold for these values on the feeder can be determined.

The calculated results for the numerical characteristics of the random variables and Pearson's chi-squared test values are shown in Table 4. The theoretical curves of the Gaussian distribution are plotted in Fig. 2 for voltage and in Fig. 3 for frequency.

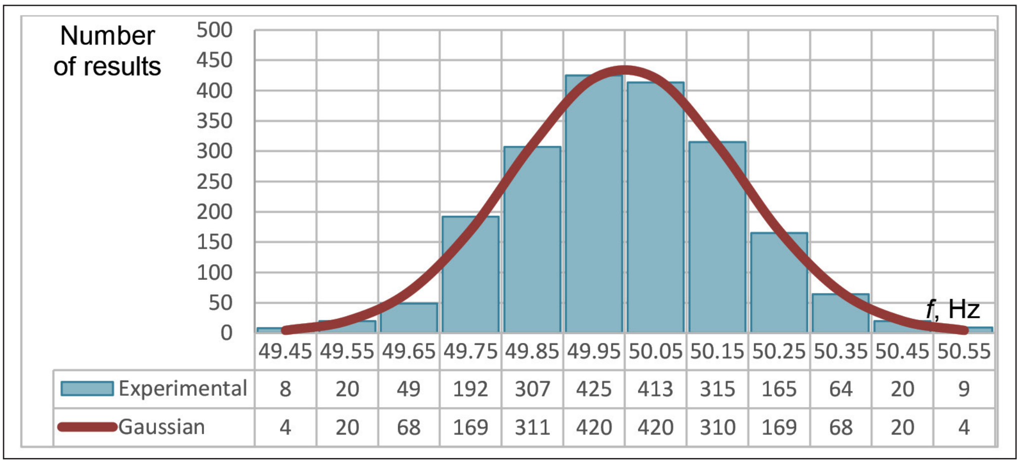
ESTIMATION OF THE VOLTAGE AND FREQUENCY VARIATIONS INFLUENCE ON THE RLC-LOAD-CIRCUIT PARAMETERS

Let us assume that the same voltage and frequency distributions take place in a conventional single-phase power network and find out how the voltage and frequency deviations from their



	189.8	195.3	200.8	206.3	211.8	217.3	222.8	228.3	233.8	239.3	244.8	250.3
Experimental	10	32	92	322	684	882	958	720	392	129	37	8
Gaussian	5	28	112	318	643	929	960	708	373	141	38	7

Fig. 2. Probability density function (blue (online) bar chart) and normal distribution (red (online) line) of voltage



	49.45	49.55	49.65	49.75	49.85	49.95	50.05	50.15	50.25	50.35	50.45	50.55
Experimental	8	20	49	192	307	425	413	315	165	64	20	9
Gaussian	4	20	68	169	311	420	420	310	169	68	20	4

Fig. 3. Probability density function (blue (online) bar chart) and normal distribution (red (online) ine) of frequency

Table 4. **Numerical characteristics of the random *U* and *f* and the Pearson's chi-squared test values**

System's parameter	m_x, [V] or [Hz]	D^*, [V^2] or [Hz^2]	σ^*, [V] or [Hz]	l	k_x	α	$\chi^2_{\exp(X)}$	$\chi^2_{cr(X)}$
U	220.52602	90.01576	9.48766	12	9	0.05	16.05771	16.91898
f	49.99982	0.03297	0.18159	12	9	0.05	16.58055	16.91898

rated values (220 V and 50 Hz) effect the network parameters and the readings of a power meter. Let us consider that an RLC-load, Fig. 4, is connected to such the power network with supply voltage U = 220 V, and the RLC circuit parameters are the following: resistance R = 21.0027 Ω, inductance L = 55.42 mH, capacitance $C = 10^3$ μF.

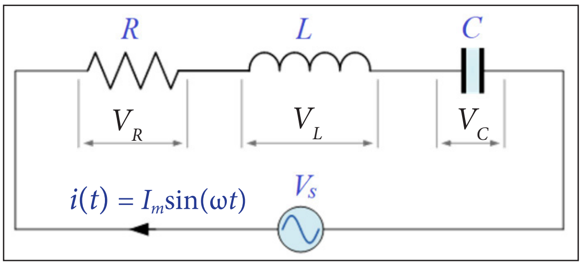


Fig. 4. The series RLC-load-circuit under study

Voltage and frequency variations in the RLC circuit will affect the values of active and reactive power, active and reactive energy, and current. These quantities can be determined by the following formulas:

Active power of the fundamental harmonic

$$P_{(1)} = U_{(1)} \cdot I_{(1)} \cdot \cos\varphi_{(1)}, \tag{17}$$

where $U_{(1)}$ is the rms voltage value of the fundamental harmonic, $I_{(1)}$ is the rms current value of the fundamental harmonic, and $\cos\varphi_{(1)}$ is the power factor (PF) of the fundamental harmonic.

Reactive power of the fundamental harmonic

$$Q = U_{(1)} \cdot I_{(1)} \cdot \sin\varphi_{(1)}. \tag{18}$$

Since electric energy consumed in the network is proportional to power $WP \equiv P$, $WQ \equiv Q$ [12], then the formulas for active and reactive electric energy are

$$W_P = P_{(1)} \cdot t, \tag{19}$$

where t is the time, and

$$W_Q = Q_{(1)} \cdot t. \tag{20}$$

Here and after it is assumed that the voltage signal does not contain any higher harmonics, subharmonics or interharmonics. If the voltage and current are of a proper sinusoidal waveform, then the readings of electric energy meters will correspond to the formulas (19, 20). Hence, this is not the case when harmonic distortions are present in the power network [13, 14].

The PF in (17) can be defined as

$$\cos\varphi = \frac{R}{\sqrt{R^2 + \left(2\pi f L - \dfrac{1}{2\pi f C}\right)^2}}, \tag{21}$$

where R is the resistance, L is the inductance, and C is the capacitance of the electric circuit. The expression $2\pi fL$ in (20) and (21) corresponds to the inductive circuit reactance X_L [Ω], and the expression $1/2\pi fC$ corresponds to the conductive circuit reactance X_C [Ω]. Reactances depend on the values of inductance L, capacitance C and frequency in the distributional network f.

The value of sinφ in (18) can be defined as

$$\sin\varphi = \frac{2\pi f L - \dfrac{1}{2\pi f C}}{\sqrt{R^2 + \left(2\pi f L - \dfrac{1}{2\pi f C}\right)^2}}. \tag{22}$$

The current value is to be determined by the Ohm's law

$$I = \frac{U}{\sqrt{(R^2 + (X_L - X_C)^2}}. \tag{23}$$

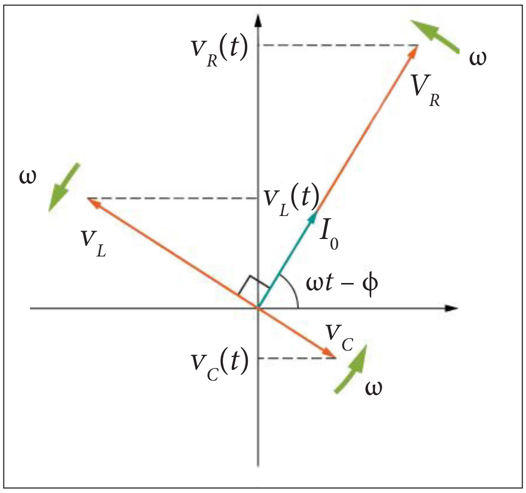


Fig. 5. The principal phasor diagram for the RLC series circuit

The principal phasor diagram for the RLC-load series circuit is shown in Fig. 5. The size of the reactive voltage vector, the PF and the phase shift between current and voltage depend on the nature of the reactance. The phasor diagram demonstrates the situation when $X_L > X_C$.

To determine the effect of voltage and frequency variations on the parameters of the electric circuit in Fig. 4, the C program code was created in Borland C++ 3.1, according to the methodology given in [15].

Sequence of voltage deviations simulation in Borland C++ 3.1

1. Consider the cases when the voltage deviates within the following limits regarding U_{rated}: ±2.5%, ±5%, ±7.5%, ±10%, ±12.5% and ±15%.
2. Determine the values of the variance and the standard deviation of voltage by (9)–(13). The mean expectation, variance, standard deviation of voltage and parameters of the electric circuit are entered in the software.
3. For frequency the median $m_f = 49.9998$ Hz and the standard deviation $\sigma_f = 0$ are assumed (i.e. the frequency is considered constant). Input these data into the program.
4. The impedance of the electric circuit is considered to be independent from voltage, while reactance is frequency-dependent.
5. The number of calculated iterations to find a random variable: 10^6.

The calculated results are presented in Table 5, and the corresponding dependency graphs are shown in Fig. 6.

It is clear from the graphs of Fig. 6 that voltage variation has a significant effect on active and reactive power, and current in the RLC-load. Voltage increase in the electrical network causes an increase in active and reactive power consumption and leads to higher current flows, while voltage decrease has a reverse effect on the named values. According to (19) and (20), voltage variations have a similar effect on the readings of active and reactive power meters, under the assumption that voltage deviations in the ranges specified in Table 5 do not affect

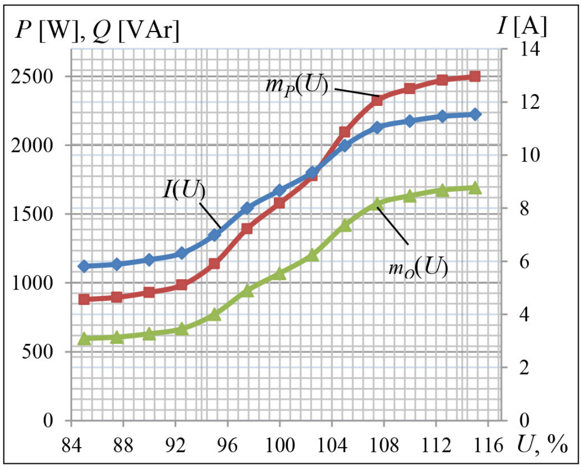


Fig. 6. Dependency graphs for $m_P(U)$, $m_Q(U)$ and $I_m(U)$

Table 5. **The results of voltage variation simulation**

Calculation results			Results of simulation								
U_{rated}, %	D_U, V²	σ_U, V	cosφ	sinφ	I, A	X_L, Ω	X_C, Ω	m_P, W	σ_P, W	m_Q, VAr	σ_Q, VAr
85	88.43176	9.4038	0.827919	0.560847	5.81233	17.4107	3.1831	879.413	141.853	595.73	96.094
87.5	83.82168	9.1554	0.827919	0.560847	5.88788	17.4107	3.1831	895.092	139.344	606.352	95.3944
90	73.81909	8.5918	0.827919	0.560847	6.05928	17.4107	3.1831	931.235	133.404	630.835	90.3704
92.5	60.51496	7.7791	0.827919	0.560847	6.30645	17.4107	3.1831	984.736	124.233	667.078	84.1578
95	30.7723	5.5473	0.827919	0.560847	6.98521	17.4107	3.1831	1140.07	95.3574	772.305	64.5968
97.5	4.971675	2.2297	0.827919	0.560847	7.9942	17.4107	3.1831	1393.77	42.3842	944.168	28.7118
100	0	0	0.827919	0.560847	8.67232	17.4107	3.1831	1579.6	0.000688	1070.05	0.000594
102.5	4.971675	2.2297	0.827919	0.560847	9.35045	17.4107	3.1831	1777.73	47.8545	1204.27	32.4175
105	30.7723	5.5473	0.827919	0.560847	10.3594	17.4107	3.1831	2095.32	129.208	1419.41	87.5277
107.5	60.51496	7.7791	0.827919	0.560847	11.0382	17.4107	3.1831	2324.3	190.779	1574.52	129.237
110	73.81909	8.5918	0.827919	0.560847	11.2854	17.4107	3.1831	2410.75	214.569	1633.08	145.353
112.5	83.82168	9.1554	0.827919	0.560847	11.4568	17.4107	3.1831	2471.66	231.496	1674.34	156.82
115	88.43176	9.4038	0.827919	0.560847	11.5323	17.4107	3.1831	2498.75	239.069	1692.7	161.949

the accuracy of active and reactive electric energy determination by the meter. For the considered case, if the voltage is +15% of U_{rated}, then electric energy consumption (for both active W_P and reactive W_Q component) will be 158.19% higher than for 220 V supply voltage. If the voltage is –15% of its rated value, then electric energy consumption will be just 55.67% of the electric energy consumption of the 220 V supply voltage. Even small voltage variations lead to significant changes of power flow. Therefore, to optimize power consumption, power losses and equipment lifespan, voltage is desired to be as close to its rated value as possible.

Sequence of frequency deviations simulation in Borland C++ 3.1

1. Consider the cases when the frequency deviates within the following limits regarding f_{rated}: ±0.2%, ±0.4%, ±0.6%, ±0.8%, ±1% and ±1.2%.

2. Determine the values of the variance and the standard deviation of frequency by (9)–(13). The mean expectation, variance, standard deviation of frequency and parameters of the electric circuit are entered in the software.

3. For voltage the median $m_U = 219.98$ V and the standard deviation $\sigma_U = 0$ are assumed (i.e. voltage is considered constant). Input these data into the program.

4. The impedance of the electric circuit is considered to be independent from voltage, while reactance is frequency-dependent.

5. The number of calculated iterations to find a random variable: 10^6.

The obtained results are summarized in Table 6, and the corresponding dependency graphs are shown in Figs. 7 and 8.

As seen from the graphs of Figs. 7 and 8, the influence of frequency variations within the considered limits on the values of active and reactive power and other parameters of the RLC-circuit are insignificant. The slope of the graphs is very smooth and barely noticeable. Frequency increase/decrease causes very small changes in active and reactive power consumption.

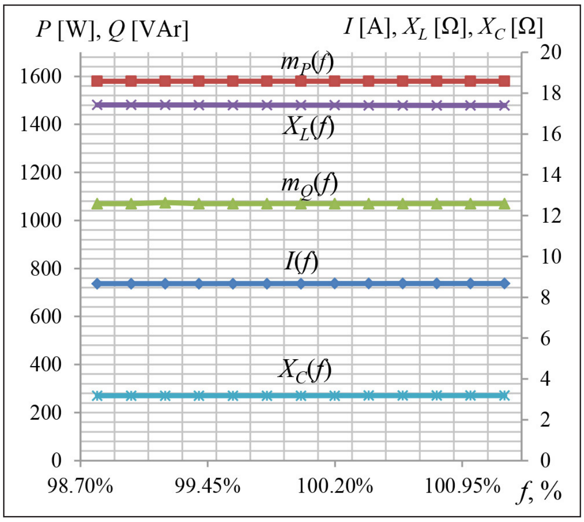


Fig. 7. Dependency graphs for $m_P(f)$, $m_Q(f)$, $I(f)$, $X_L(f)$ and $X_C(f)$

Table 6. **The results of frequency variation simulation**

Calculation results			Results of simulation								
f_{rated}, %	D_f, Hz^2	σ_f, Hz	cosφ	sinφ	I, A	X_L, Ω	X_C, Ω	m_P, W	σ_P, W	m_Q, VAr	σ_Q, VAr
98.8	0.033656437	0.18346	0.827673	0.561211	8.66975	17.4221	3.18102	1579.58	5.29893	1070.04	2.11628
99	0.031759011	0.17821	0.82768	0.5612	8.66982	17.4218	3.18108	1579.59	5.1473	1070.04	2.05571
99.2	0.028191967	0.16790	0.827694	0.56118	8.66996	17.4211	3.1812	1579.59	4.849952	1074.04	1.93677
99.4	0.022126468	0.14875	0.82772	0.561142	8.67023	17.4199	3.18141	1579.59	4.29643	1070.04	1.71587
99.6	0.009665499	0.09831	0.827787	0.561042	8.67094	17.4168	3.18199	1579.59	2.83958	1070.05	1.13403
99.8	0.001507312	0.03882	0.827867	0.560924	8.67178	17.4131	3.18266	1579.59	1.12129	1070.05	0.447814
100	0	0	0.827919	0.560847	8.67232	17.4107	3.1831	1579.6	0.000688	1070.05	0.000594
100.2	0.001507312	0.03882	0.827971	0.56077	8.67287	17.4083	3.18354	1579.6	1.1213	1070.04	0.447855
100.4	0.009665499	0.09831	0.828051	0.560653	8.67371	17.4046	3.18421	1579.61	2.83967	1070.04	1.13429
100.6	0.022126468	0.14875	0.828119	0.560553	8.67442	17.4015	3.18479	1579.61	4.29665	1070.03	1.71646
100.8	0.028191967	0.16790	0.828145	0.560515	8.67468	17.4003	3.185	1579.62	4.82981	1070.03	1.93754
101	0.031759011	0.17821	0.828158	0.560494	8.67483	17.3997	3.18512	1579.62	5.14762	1070.02	2.05657
101.2	0.033656437	0.18346	0.828165	0.560484	8.6749	17.3993	3.18518	1579.62	5.29927	1070.02	2.11719

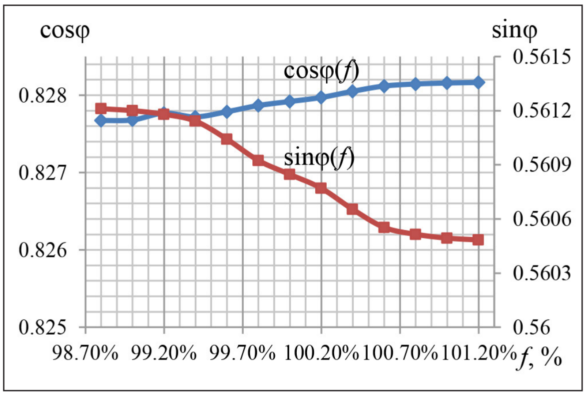


Fig. 8. Dependency graphs for cosφ(f) and sinφ(f)

According to formulas (19) and (20), frequency variations within the considered range will hardly change the readings of active and reactive power meters. For the considered example, if the frequency is +1.2% of f_{rated}, then electric energy will be just 100.0013% higher for W_P, and 99.9972% lower for W_Q, compared to the case when the frequency is 50 Hz. If the frequency is –1.2% of its rated value, then electric energy consumption will be just 99.9987% lower for W_P, and 99.9991% lower for W_Q, compared to the case when the frequency is 50 Hz.

DOES VOLTAGE ALWAYS OBEY THE GAUSSIAN DISTRIBUTION?

Although under normal operating conditions the voltage variations at the customer's side are considered to be of a stochastic character and can be described by a normal distribution [3], this may not be the rule for all consumers and for all points in the LV/MV distribution power grid. For the example discussed in the previous sections, the null hypothesizes of Gaussian distribution of voltage and frequency were validated and accepted. However, the distribution laws of these quantities in the LV or MV power networks may be different, e.g. hypergeometric, binominal, negative binomial, log-normal, chi-squared, Poisson, Student's, Cauchy, Weibull, gamma or other.

To check this statement, five series of measurements of the voltage profile were taken at the university facility. The measurements were taken for three workdays (Monday, Wednesday, Friday) and for weekend. 18 000 voltage measurements were made for each sample at 1 s step, using a power quality analyzer ENA330, Class A (Czech Republic). The data were processed according to the above described method, using the numerical characteristics of the random variables and their systems, and the Pearson's chi-squared test was made to verify whether the voltage profiles obey the normal distribution law. The results of the calculations are summarized in Table 7. The $f(U)$ dependences are demonstrated in Fig. 9.

For all the five cases, the Pearson's chi-squared test condition $\chi^2_{\exp(x)} < \chi^2_{ecr(x)}$ is not satisfied, which means that the null hypothesis of the normal distribution of random voltage at the university's grid should be rejected. In other words, the assumed normal distribution poorly describes the empirical data. Fig. 9(a–e) also reveals noticeable disparities between the theoretical Gaussian curve and the graphs plotted from the experimental results.

CONCLUSIONS

Power quality is characterized by the voltage quality and the frequency quality of AC voltage. In this paper, the undesired impacts of voltage and frequency variations and control strategies to maintain these PQ indicators at the appropriate levels are analysed. Requirements of

Table 7. **Numerical characteristics of the random U and the Pearson's chi-squared test results**

Sample No.	m_U, [V]	D^*_U, [V²]	σ^*_U, [V]	$\chi^2_{\exp(U)}$	$\chi^2_{ecr(U)}$
1	226.62891	0.31456	0.56086	413.95754	21.02607
2	228.79213	0.7718	0.87852	4352.56972	21.02607
3	228.73425	0.16745	0.40921	1659.83991	21.02607
4	227.38246	0.53085	0.72859	1796.04649	21.02607
5	229.56258	0.14817	0.38493	1177.21162	21.02607

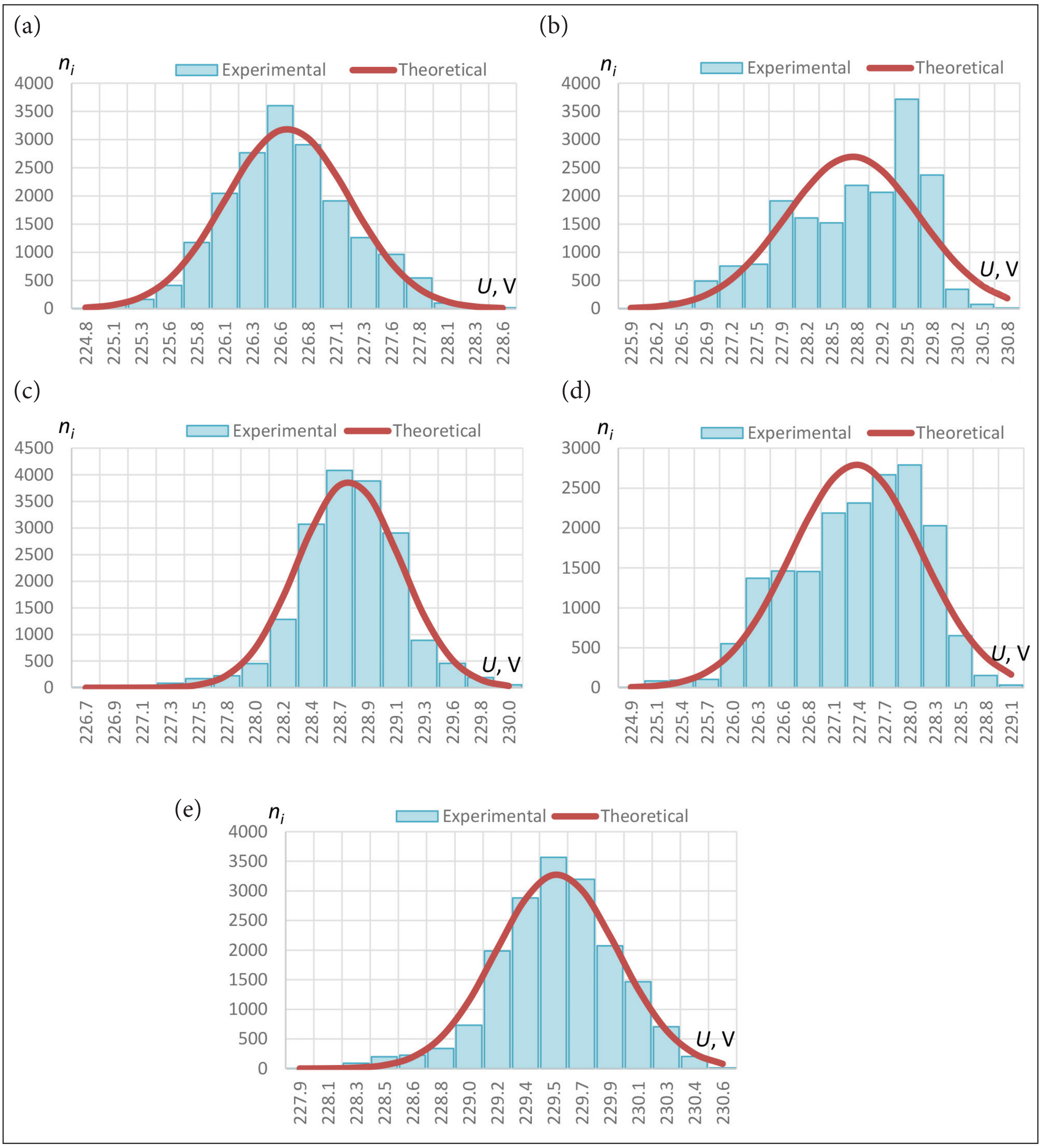


Fig. 9. Probability density function (blue (online) bar chart) and normal distribution (red (online) line) of voltage for (a) workday (Monday), (b) workday (Wednesday), (c) workday (Friday), (d) weekend (Saturday) and (e) weekend (Sunday)

the EN 50160 standard for PQ regulation, valid in Ukraine and the EU, are also considered. The results of voltage and frequency measurements were processed using the numerical characteristics of random variables and their systems, and the hypothesizes of Gaussian distribution of voltage and frequency were validated and accepted. Assuming that similar voltage and frequency profiles take place in a single-phase LV distribution network, it was evaluated how their deviations from the rated values (220 V and 50 Hz) effect the network parameters and the readings of a power meter. The load was represented as a series RLC-circuit.

Voltage profile is of great importance to consumers because it is a basic demand for electrical equipment running near the rated voltage. Even small voltage variations significantly change the power flow. For the considered case, if the voltage is +15% of U_{rated}, then electric energy consumption, for both W_P and W_Q components, will be 1.6 times higher, compared to the case when the supply voltage rms value is 220 V. If the voltage is −15% of its rated value, then electric energy consumption will be 1.8 times lower, compared to the 220 V case. Thus, to optimize power consumption, power losses and equipment lifespan, it is desirable to maintain voltage as close to its rated value as possible. For the considered case study, frequency deviations within the range ±1.2% of f_{rated} demonstrated a negligible influence on the RLC-load-circuit parameters and the theoretical readings of active and reactive power meters. Although the frequency variation seems to be not critical, it is important to maintain its value within the acceptable limits, since the frequency deviations worsen the operation of asynchronous motors and other equipment. Thus, the limits given in EN 50160-2010 should be respected.

Under normal operating conditions, the voltage variations at the customer's side have a stochastic nature. For some cases, the voltage profile obeys the normal distribution. However, distribution of the random variable (voltage) depends on the type of load and the point of measurement in the LV and MV distribution power systems, which means that its distribution law should be defined for each case separately.

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MAITINIMO ĮTAMPOS IR DAŽNIO VARIACIJOS ĮTAKA ELEKTROS ĮRANGAI BEI ŽEMOS IR VIDUTINĖS ĮTAMPOS TINKLŲ APKROVOMS

Santrauka

Straipsnyje analizuojama maitinimo įtampa ir dažnio kokybė žemos ir vidutinės įtampų skirstomuosiuose tinkluose. Įtampos ir dažnio variacijų, galiojančių nepažeidžiant nustatytų standartų ribų, įtaka elektros tinklo parametrams buvo ištirta remiantis tikimybių teorija ir statistika. Įvertinimui naudota RLC tipo grandinė su apkrova. Buvo sudarytas įtampos ir dažnio variacijų skirstinys ir pasiūlytos optimalios įtampos ir dažnio kokybės parametrų vertės, atsižvelgiant į energijos naudojimą, tinklo nuostolius ir elektros įrangos gyvavimo laiką.

Raktažodžiai: įtampos variacijos, dažnio variacijos, energijos kokybė, elektros energijos skaitiklis, normalusis skirstinys, Pirsono (Chi kvadratu) kriterijus, elektros skirstomasis tinklas